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Riemannian and Hermitian geometry of invariant structures on homogeneous manifolds

Abstract: We intend to present a brief observation of recent results on invariant affinor and (pseudo)Riemannian structures on homogeneous manifolds. Some basic notions, results and references can be found in [1-5].

Invariant *metric f -structures* ($f^3 + f = 0$, K.Yano) on naturally reductive homogeneous spaces were considered. We indicate the conditions under which these f -structures belong to the main classes of *generalized Hermitian geometry* such as *Kähler* (Kf), *Hermitian* (Hf), *Killing* ($Killf$), *nearly Kähler* (NKf), G_1f -*structures* (G_1f). As a particular case, it follows some results of V.F.Kirichenko, E.Abbena and S.Garbiero in Hermitian geometry. As is known, *canonical f -structures on homogeneous k -symmetric spaces* provide a remarkable vast class of invariant metric f -structures including almost Hermitian structures. First, canonical f -structures on homogeneous 4- and 5-symmetric spaces were completely characterized in the sense of generalized Hermitian geometry. Besides, partial results were also obtained for the cases $k = 4n, n \geq 1$ and some others. Moreover, recently four canonical f -structures on homogeneous 6-symmetric spaces were also completely studied in this respect. Many particular examples with respect to families of invariant (pseudo)Riemannian metrics were investigated in detail. They are the flag manifolds $SU(3)/T_{max}$, $Sp(3)/Sp(1) \times Sp(1) \times Sp(1)$, $SO(n)/SO(2) \times SO(n-3)$, $n \geq 4$, the Stiefel manifold $SO(4)/SO(2)$ and some others. Specifically, we present invariant Killing f -structures with non-naturally reductive metrics as well as construct invariant Kähler f -structures on some naturally reductive not locally symmetric homogeneous spaces.

The classification of three-dimensional Lie groups admitting a left-invariant Lorentzian metric and an almost harmonic Schouten-Weyl tensor was obtained. We introduce a skew-symmetric 2-tensor using the contraction of the Schouten-Weyl tensor in the direction of an arbitrary vector. The structure of three-dimensional Lie groups and Lie algebras endowed with a left-invariant Riemannian metric and a harmonic skew-symmetric 2-tensor above mentioned was investigated. As a result, the complete classification of such Lie groups and Lie algebras in directions of harmonic vectors was obtained. Besides, three-dimensional metric Einstein algebras for three-dimensional Lie groups with left-invariant Lorentzian metrics were also found.

Some formulas for calculating the Schouten-Weyl tensor and the Weyl tensor in terms of the structural constants of the metric triple Lie algebras of a homogeneous Riemannian space were obtained. Almost harmonic and harmonic homogeneous Riemannian metrics on some particular classes of homogeneous spaces were found. Invariant metrics with the harmonic Weyl tensor on the generalized Berger-Wallach spaces were also investigated.

Finally, theorems of general character on invariant pseudo-Riemannian metrics constructed by means of canonical invariant structures on generalized symmetric spaces were proved. We illustrate here a fundamental role of the canon-

ical almost product and almost complex structures on homogeneous k -symmetric spaces.

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References

1. Balashchenko V.V. Invariant f -structures in generalized Hermitian geometry // Proc. of the Conf. "Contemporary Geometry and Related Topics". Belgrade, Faculty of Mathematics, University of Belgrade. 2006. P.5-27.
2. Besse A. L. Einstein manifolds, Springer-Verlag, Berlin, Heidelberg, 1987.
3. Kowalski O. Generalized symmetric spaces, LN in Math., V.805, Berlin, Heidelberg, New York: Springer-Verlag, 1980.
4. Nikonorov Yu.G., Rodionov E.D., Slavsky V.V. Geometry of homogeneous Riemannian manifolds // Modern Math. and its Appl. 2006. V.37. P.1-78.
5. Wang M. Einstein metrics from symmetry and bundle constructions // Surv. Diff. Geom.: Essays on Einstein manifolds.- Cambridge: International Press, 1999. P.287-325.