Cemile Elvan Dinc (Kadir Has University, Istanbul, Turkey)
Sezgin Altay, **Fusun Ozen**
Kahler spaces with recurrent H-projective curvature tensor

`Abstract:` A Kahler space K_n is a Riemannian space in which, along with the metric tensor $g_{ij}(x)$, and affine structure $F_i^h(x)$ is defined that satisfies the relations

$$F_\alpha^h F_i^\alpha = e\delta_i^h \ ; \quad F_{(i}^\alpha g_{j)\alpha} = 0 \ ; \quad F_{i,j}^h = 0, \quad where \quad e = \pm 1, 0.$$

Here and in what follows $''{,}''$ denotes a covariant derivative.

If $e = -1$, then K_n is an elliptically Kahler space K_n^-, if $e = +1$, then K_n is a hyperbolically Kahler space K_n^+, and if $e = 0$ and $R_g\|F_i^h\| = m \leq \frac{n}{2}$, then K_n is an m-parabolically Kahler space $K_n^{0(m)}$. Necessarily, the spaces K_n^+, K_n^- and K_n^0 are of an even dimension.

The spaces K_n^- were first considered by P. A. Shirokov in 1966.

In this paper, we consider the holomorphically projective curvature tensor in the space K_n^-. Problems containing in this study are as follows:

Every H- projective recurrent Kahler space is recurrent. It' s shown that the curvature tensor of H- projective recurrent Kahler- Einstein space is harmonic and if this space with constant curvature has positive definite metric then it's flat.

The notion of quasi- Einstein space was introduced by M. C. Chaki and R. K. Maity in 2000. According to him a non- flat Riemannian space $(n > 2)$ is said to be a quasi- Einstein space if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies the condition

$$S(X, Y) = ag(X, Y) + bA(X)A(Y),$$

where a and b are scalars of which $b \neq 0$. A is a nonzero 1- form such that $g(X, \rho) = A(X)$ for all vector fields X and ρ is an unit vector field. We searched relations between the quasi- Einstein spaces and H- projective recurrent Kahler spaces; and then noticed some relations between these two spaces supporting a theorem.

Subsequently, we obtained a relation between the Ricci tensor and the curvature tensor by defining a special vector field $u_i = \lambda_s F_i^s$ in a positive definite Kahler space with non- constant scalar curvature and also it is shown that the ricci tensor' s rank is two and its eigenvalues are zero and $\dfrac{R}{2}$.

On the other hand, we proved that the length of the special vectors defined by the recurrence vector λ_l are constant in a H- projective Kahler space with non- zero scalar curvature.

In the last section, we presented an example of H- projective recurrent Kahler space. For this, H- projective recurrent Kahler space is taken as the subset $M \subset R^4$ where M is an open connected subset of R^4 and (x^1, x^2, x^3, x^4) are the Cartesian coordinates in R^4. For the metric tensor g_{ij} and the tensor field F_j^h given in our subsequent work, using the recurrency condition

$P_{hijk,l} = \lambda_l P_{hijk}$, we have a linear first order partial differential equation. By solving the Lagrange system, the general solution of $\varphi(x^1, x^3)$ is found where the function φ is one of the components of the metric tensor of H- projective recurrent Kahler space. In this case, the line element of this space is determined. We also want to mention that the open connected subset $M \subset R^4$ admits a almost L structure.

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Key Words : Kahler manifold, recurrent space, special quasi Einstein manifold, Ricci tensor, Holomorphically projective curvature tensor.

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