Maria Dyachkova (Kazan State University, Kazan, Russia)
On Hopf bundle analogue for semiquaternion algebra

`Abstract:` The subject of the talk is to study the principle algebra bundle by it's subalgebra. Let $\mathfrak{A}$ be a 4-dimensional associative unital nonreducible semiquaternion algebra [1]. The basic units multiply by the following rules: $e_1^2 = -1,\ e_1e_2 = -e_2e_1 = e_3,\ e_1e_3 = -e_3e_1 = -e_2,\ e_2^2 = e_3^2 = e_2e_3 = e_3e_2 = 0$. Consider the degenerate scalar product:$(x, y) = x_0y_0 + x_1y_1$. Thus the algebra $\mathfrak{A}$ has the 4-dimensional semieuclidean space structure ${}_2\mathbb{R}^4$ with rank 2 semimetric.

The set G consisting of invertible elements of $\mathfrak{A}$ is a Lie group, which is connected but not simply connected. This algebra contains 2-dimensional subalgebras isomorphic either to complex or dual number algebra. Let H be the set of invertible elements of dual algebra with the basis $\{1, e_2\}$. H is Lie subgroup of G. A semiquaternion can be reprezented in the following way: $x = z_1 + e_1z_2$, here $z_1 = x_0 + x_2e_2$, $z_2 = x_1 + x_3e_2$.

The bundle $(G, \pi, G/H)$ of right cosets is a principle locally trivial bundle with the structure group H. The base is the subset $M = \{[z_1 : z_2] \in P(e_2)||x|^2 \neq 0\}$ of projective line $P(e_2)$ over the dual algebra without a point. The canonical projection is $\pi(x) = (\overline{z}_2 : z_1)$.

The set of invertible elements with the module one (the *semieuclidian sphere* ${}_2S^3 = \{x \in \mathfrak{A}||x|^2 = 1\}$) is the 2-fold cover of the special orthogonal group $SO(3)$. It is an analogue of the Hopf bundle.

Consider the restriction of the bundle $(G, \pi, G/H)$ to the irreducible unit sphere ${}_2S^3$, i.e. the bundle $\pi :_2 S^3 \to M$. Then the restriction of the group H to ${}_2S^3$ is the Lie group S. The bundle $({}_2S^3, \pi, M)$ is principle locally trivial with total space and fiber being Lie groups ${}_2S^3$ and S, respectively .

References

[1] B. Rosenfeld, *Geometry of Lie Groups*, Dodrecht-Boston-London, Kluwer, 1997.