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Existence of Ehresmann connection on manifold foliated by locally free action of Lie group

`Abstract:` Recall first the following proposition [3]:

Theorem 1.*Let a manifold M with foliation F meet the following conditions:*

1) $\mathrm{codim} F = 1$.

2) *The foliation F is generated by locally free action of $H \cong \mathbb{R}^n$.*

3) *There exists a compact connected transversal on (M, F).*

Then there exist a transversal homotopic to H-complementary one and an Ehresmann connection on (M, F).

The present work is dedicated to generalisation of this theorem to the case of noncommutative Lie groups. Note that structure of proper group actions on spaces of nonpositive Alexandrov curvature as described in [2] in this case is more specific, i.e. closedness of the transversal implies that there is no purely virtually abelian factor.

Moreover the existence of the maximal tori for any Lie group [1] allows us to prove the following statement:

Theorem 2.*Let (M, F) be manifold with codimension 1 foliation generated by action of the compact Lie group G. Let (M, F) possess compact complete transversal P. Then there exists Ehresmann connection on (M, F).*

Note that the connection constructed in this statement is not necessarily G-complemetary.

Next recall the topological (Maltsev or Iwasawa) representation of the arbitrary Lie group as product $H = G \times E$ (it is not group homomorphism). Here G is maximal compact subgroup of H and E is Euclidean space. Thus the next task is to describe foliations with leaves covered by E.

Statement 1. *Assume that the foliated manifold (M, F) is such that*

1) Each leaf $L \in F$ is covered by E and is a topological group.

2) The foliation F is transversally orientable.

3) There exists a complete transversal P on (M, F).

Then there exists an Ehresmann connection on (M, F).

References

[1] N. Bourbaki, Groupes et Algébres de Lie, Chapitre 9, Masson, 1982.

[2] P.E. Caprace, Amenable groups and Hadamard spaces with a totally disconnected isometry group, www.arXiv.org/math.GR/0705.1980v1

[3] P. N. Ivanshin, Ehresmann connection on foliations generated by R^n // Differential Geometry - Dynamical Systems, Volume 9 (2007), Pp. 58-81.