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Connections on principal prolongations of principal bundles

`Abstract:` We study principal connections on the r-th order principal prolongation W^rP of a principal bundle $P(M,G)$, $\dim M = m$. The bundle $W^rP \to M$, which is a principal bundle with structure group W^r_mG, plays a fundamental role both in the geometric theory of jet prolongations of associated bundles and in the gauge theories of mathematical physics. If $G = \{e\}$ is the one-element group, then $W^r(M \times \{e\}) = P^rM$ is the r-th order frame bundle of the base M. So some our results can be viewed as a generalization of the theory of connections on P^rM. In particular, there is a canonical $\mathbb{R}^m \times \mathrm{Lie}\,(W^{r-1}_mG)$-valued 1-form Θ_r on W^rP. The torsion of a connection Δ on W^rP is the covariant exterior differential $D_\Delta\Theta_r$.

Let $EP = TP/G$ be the Lie algebroid of P. Its r-jet prolongation $J^r(EP \to M)$ coincides with the Lie algebroid of W^rP. We start from the fact that the connections Δ on W^rP are in bijection with the linear splittings $\delta : TM \to J^r(EP)$. The torsion $\tau(\delta)$ of δ can be defined by means of the jet prolongation of the bracket of EP. We deduce that $D_\Delta\Theta_r$ and $\tau(\delta)$ are naturally equivalent. Further, analogously to the case of P^rM, the torsion free connections on W^rP are in bijection with the reductions of $W^{r+1}P$ to the subgroup $G^1_m \times G \subset W^{r+1}_mG$.

According to the general theory, every principal connection Γ on P and a linear r-th order connection $\Lambda_r : TM \to J^rTM$ induce, by means of flows, a connection $\mathcal{W}^r(\Gamma, \Lambda_r)$ on W^rP. We clarify that $\mathcal{W}^r(\Gamma, \Lambda)$ can be easily constructed in the algebroid form. This enables us to deduce several original geometric results. – We also extend some our constructions and results to an arbitrary fiber product preserving bundle functor on the category of fibered manifolds with m-dimensional bases and fibered manifold morphisms with local diffeomorphisms as base map.