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Unique determination of domains

`Abstract:` This survey lecture is devoted to relatively recent results closely connected with some 200 years old classical problems.

The starting point is a familiar Cauchy theorem *about the unique determination of convex polyhedrons (in Euclidean* 3*-space* $\mathbb{R}^3$*) by their unfoldings*. Later the problems of unique determination of convex surfaces were studied by Minkowski, Hilbert, Weyl, Blashke, Cohn-Vossen and other prominent mathematicians. Yet the greatest success was achieved by A.D. Aleksandrov and his school. We want to mention the following classical theorem by A.V. Pogorelov [1]: *if two bounded closed convex surfaces in* $\mathbb{R}^3$ *are isometric in their inner metrics, then these surfaces are congruent, i.e., one of them can be translated to the other by a motion*.

A new development of the subject is due to A.P. Kopylov. Kopylov offered a new approach [2], which essentially extends the scope of the above problems. He suggested to study the unique determination of domains by relative metrics of their boundaries, i.e., the metric on the boundary is defined as a continuation of the inner metric of the domain. So, the foregoing classical problems are special cases of the problem of the unique determination of domains by relative metrics of their boundaries, namely, when the complementary sets of the domains are convex sets. Moreover, a new class of very interesting problems appears in Kopylovs approach. These new problems were studied by A.D. Aleksandrov and also by V.A. Aleksandrov, M.K. Borovikova, A.V. Kuzminykh, M.V. Korobkov and others (in this connection, see, e.g., [2][12]). They discovered the following new phenomena: domains are uniquely determined not only in the classical cases (when the complementary sets of the domains are convex bounded sets), but also when domains are convex and bounded [2]; strictly convex (A.D. Aleksandrov); bounded with piece-smooth boundary [3]; having non-empty bounded complements, where their boundaries are connected smooth $(n-1)$-manifold without edge [4]; and others.

In 2006, A.P. Kopylov considered a new unique determination problem of conformal type and proves the following assertion (see [13] and [14]): *if* $n \geq 4$*, then any bounded convex polyhedral domain* $U \subset \mathbb{R}^n$ *is uniquely determined by the relative conformal moduli of its boundary condensers in the class of all bounded convex polyhedral domains* $V \subset \mathbb{R}^n$.

We also discuss some problems related to the theory of the unique determination of the isometric and conformal types of domains in R^n.

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