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Like jet prolongation functors of affine bundles

`Abstract:` Let $F : \mathcal{AB}_m \to \mathcal{FM}$ be a covariant functor from the category $\mathcal{AB}_m$ of affine bundles with m-dimensional bases and affine bundle maps with local diffeomorphisms as base maps into the category $\mathcal{FM}$ of fibred manifolds and their fibred maps. Let $B_{\mathcal{AB}_m} : \mathcal{AB}_m \to \mathcal{M}f$ and $B_{\mathcal{FM}} : \mathcal{FM} \to \mathcal{M}f$ be the respective base functors.

A gauge bundle functor on $\mathcal{AB}_m$ is a functor F as above satisfying:

(i) (**Base preservation**) $B_{\mathcal{FM}} \circ F = B_{\mathcal{AB}_m}$. Hence the induced projections form a functor transformation $\pi : F \to B_{\mathcal{AB}_m}$.

(ii) (**Localization**) For every inclusion of an open affine subbundle $i_{E|U} : E|U \to E$, $F(E|U)$ is the restriction $\pi^{-1}(U)$ of $\pi : FE \to B_{\mathcal{AB}_m}(E)$ over U and $Fi_{E|U}$ is the inclusion $\pi^{-1}(U) \to FE$.

A gauge bundle functor F on $\mathcal{AB}_m$ is *fiber product preserving* if for every fiber product projections $E_1 \stackrel{pr_1}{\longleftarrow} E_1 \times_M E_2 \stackrel{pr_2}{\longrightarrow} E_2$ in the category $\mathcal{AB}_m$ $FE_1 \stackrel{Fpr_1}{\longleftarrow} F(E_1 \times_M E_2) \stackrel{Fpr_2}{\longrightarrow} FE_2$ are fiber product projections in the category $\mathcal{FM}$. In other words $F(E_1 \times_M E_2) = F(E_1) \times_M F(E_2)$ modulo the restriction of (Fpr_1, Fpr_2).

The most important example of fiber product preserving gauge bundle functor on $\mathcal{AB}_m$ is the r-jet prolongation functor $J^r : \mathcal{AB}_m \to \mathcal{FM}$, where for an $\mathcal{AB}_m$-object $p : E \to M$ we have $J^rE = \{j^r_x\sigma \mid \sigma$ is a local section of $E,\ x \in M\}$ and for a $\mathcal{AB}_m$-map $f : E_1 \to E_2$ covering $\underline{f} : M_1 \to M_2$ we have $J^rf : J^rE_1 \to J^rE_2$, $J^rf(j^r_x\sigma) = j^r_{\underline{f}(x)}(f \circ \sigma \circ \underline{f}^{-1})$, $j^r_x\sigma \in J^rE_1$.

The first main result in this paper is that all fiber product preserving gauge bundle functors F on $\mathcal{AB}_m$ of finite order r are in bijection with so called admissible systems, i.e. systems $(V, H, t, \mathbf{1})$, where V is a finite dimensional vector space over $\mathbb{R}$, $\mathbf{1} \in V$ is an element, $H : G^r_m \to GL(V)$ is a smooth group homomorphism from $G^r_m = invJ^r_0(\mathbb{R}^m, \mathbb{R}^m)_0$ into $GL(V)$ with $H(\xi)(\mathbf{1}) = \mathbf{1}$ for any $\xi \in G^r_m$, $t : \mathcal{D}^r_m \to gl(V)$ is a G^r_m-equivariant unity preserving associative algebra homomorphism from $\mathcal{D}^r_m = J^r_0(\mathbb{R}^m, \mathbb{R})$ into $gl(V)$.

The second main result is that any fiber product preserving gauge bundle functor on $\mathcal{AB}_m$ is of finite order.