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Submanifolds in Pseudo-Euclidean Spaces, Associativity Equations in 2D Topological Quantum Field Theories, and Frobenius Manifolds

Abstract: We prove that the associativity equations of two-dimensional topological quantum field theories are very natural reductions of the fundamental nonlinear equations of the theory of submanifolds in pseudo-Euclidean spaces (namely, the Gauss equations, the Peterson–Codazzi–Mainardi equations and the Ricci equations) and determine a natural class of *potential* flat submanifolds without torsion. We show that all potential flat torsionless submanifolds in pseudo-Euclidean spaces possess natural structures of Frobenius algebras on their tangent spaces. These Frobenius structures are generated by the corresponding flat first fundamental form and the set of the second fundamental forms of the submanifolds (in fact, the structural constants are given by the set of the Weingarten operators of the submanifolds). We prove that each N-dimensional Frobenius manifold can locally be represented as a potential flat torsionless submanifold in a $2N$-dimensional pseudo-Euclidean space. By our construction this submanifold is uniquely determined up to motions. Moreover, we consider a nonlinear system, which is a natural generalization of the associativity equations, namely, the system describing the class of all flat torsionless submanifolds in pseudo-Euclidean spaces, and prove that this system is integrable by the inverse scattering method.