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On the conditions of the existense of Backlund maps and transformations for the third order evolution-type PDEs with one dimensional variable

`Abstract:` The work is contributed to the geometrical theory of Backlund transformations (BT) for the third order PDE's of the evolution type with one dimensional variable:

$$z_t - f(t, x, z, z_x, z_{xx}, z_{xxx}) = 0 \tag{1}$$

See [1, 2] on the geometrical theory of BT.

Followinf F.Pirani and D.Robinson [3], we treat the concept of the Backlund Transformations (BT) as a particular case of more general concept – Backlund Map (BM), which we understand as a connection, defining zero-curvature representation for the given evolution equation.

It is proved, that equation (1) possess BM if and only if it has a form:

$$z_t + K(z,\, z_x,\, z_{xx}) \cdot z_{xxx} + L(z,\, z_x,\, z_{xx}) = 0 \tag{2}$$

The particular case of the equation (2) are the equations of the form: $z_t + z_{xxx} + M(z, z_x) = 0$. It is proved that these equations (2) posses BM if and only if they have a form:

$$z_t + z_{xxx} + \beta(z)(z_x)^3 + \alpha(z) \cdot z_x = 0$$

Among these equations are the equations of the form:

$$z_t + z_{xxx} + \alpha(z) \cdot z_x = 0 \tag{3}$$

One of them is a well-known KdF-equation $z_t - 6z \cdot z_x - z_{xxx} = 0$. It is proved, that equations (3) posses BT if and only if they have a form:

$$z_t + z_{xxx} + \frac{\varphi(z)}{az + b} \cdot z_x = 0 \tag{4}$$

The system of PDEs, which define BT (so called Backlund system) is given.

LITERATURE.

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[3] Pirani F.A.E., Robinson D.C., Sur la definition des transformations de Backlund. // C.R.Acad.Sc. Paris; Serie A. – T.285 – P.581-583