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Torse-forming vector field in a pseudo-Ricci symmetric space

Abstract: A non-flat Riemannian space is called pseudo-Ricci symmetric and denoted by $(PRS)_n$ if the Ricci tensor is non-zero and satisfies the condition $R_{ij,k} = 2\lambda_k R_{ij} + \lambda_i R_{kj} + \lambda_j R_{ik}$ where λ_i is covariant vector(non- zero simultaneously). Pseudo-Ricci symmetric space introduced by Chaki,[1], before.

In this paper, we shall consider a torse-forming vector field v^i in pseudo-Ricci symmetric space. We may assume that

$$v^h_{,i} = \phi_i v^h + \rho\delta^h_i \qquad (1.1)$$

where ρ and ϕ are any scalar function and covariant vector field, respectively, [2], [3]. Let us take an innitesimal vector field $\bar{x} = x^i + v^i(x)\delta t$ in $(PRS)_n$ where $v^i(x)$ is a covariant vector field.

We prove that if $(PRS)_n$ with positive definite metric and has scalar curvature non-constant admits an infinitesimal pseudo-homothetic motion, then this is either a homothetic motion or a motion. If the equation (1.1) satisfy the relation $\nabla_i v^h = \rho\delta^h_i$ then this vector field is called concurrent.

Some theorems that was prove in this paper are in the following: We consider a pseudo-Ricci symmetric space (V^n, g), has positive definite metric and non-constant scalar curvature, admits a pseudo-homothetic motion.

THEOREM 1. Let (V^n, g) admit a recurrent vector field. In order that the motion be isometry, it is necessary and sucient that the vector v and λ be orthogonal to each other.

THEOREM 2. Let (V^n, g) admit a concurrent vector field. If the vector v and λ be orthogonal to each other then V_n whose streamlines are geodesics can not exist.

THEOREM 3. Let (V^n, g) admit a recurrent vector field. Then it will be isometry and the vectors v and λ are orthogonal.

THEOREM 4. Let (V^n, g) admit a concircular vector field. In order that the vector ρ_j and v_j be collinear, it is necessary and sufficient that ϕ_j and v_j be collinear. If $\varphi = \rho$ then this vector field is either recurrent or concurrent.

THEOREM 5. Let (V^n, g) admit a torse-forming vector field. In order that the vector v_i and ϕ_i be orthogonal, it is necessary and sufficient that $\varphi = \rho$.

THEOREM 6. Let (V^n, g) admit a semitorse-forming vector field. In order that the vector v_k and ϕ_k be orthogonal, it is necessary and sufficient that v_k and ρ_k be orhogonal.

References

[1] M. C. Chaki, On contact metric R-harmonic manifolds, *An. Stiin. Ale Univ. AL. I. Cuza, lasi* **33**, (1987), 53-58.

[2] Yano K. , On torse forming directions in Rieamannian spaces. *Proc. Imp. Acad. Tokyo* **20**,(1944), 701-705.

[3] Josef Mikes, Lukas Rachunek , Torse-forming vector fields in T-semi-symmetric Riemannian spaces *Steps in Differential Geometry, Proceedings of Colloquium on Differential Geometry*, 25-30 July, 2000,Debrecen, Hungary.

[4] Lukas Rachunek, Josef Mikes On semi-torse-forming vector fields 3^{rd} *International Conference APLIMAT 2004* , 835-840.