

Alexander M. Shelekhov (Tver State University, Tver, Russia)

Triangulations of plane and sphere by 3 pencils of circles

Abstract: We say that a plane π is triangulated by 3 smooth foliations λ_i , $i = 1, 2, 3$, if the 3-web W formed by λ_i is hexagonal, i.e. All hexagonal configurations are closed on W (see. Fig.1). By W. Blaschke [1], 3-web W is hexagonal if it is regular, that is equivalent to a 3-web formed by 3 families of parallel straight lines.

About 1953, W. Blaschke stated the problem to find all hexagonal (regular) 3-webs formed by 3 pencils of circles on a plane or a sphere (we call such webs circle-webs). We give a complete and transparent solution to the problem. The solution is based on the following proposition that we call the theorem on the boundaries: each smooth boundary of a regular curvilinear 3-web can be only a leaf of this web. (The boundary consists of the points in which the leaves of one of foliations of 3-web W tangent to the leaves of another foliation of W). Generally, the boundary of an arbitrary circle web is a 4-degree algebraic curve. By the theorem on the boundaries this algebraic curve is decomposed in two circles for a regular circle web W .

Using Darboux transformation we find all the cases when the above mentioned 4-degree curve is a decomposition of 2 circles belonging to 3-web W . The main result is the following: only 8 classes of hexagonal circle webs indicated in [2] exist there.

Detailed proofs see in [3].

[1]. W. Blaschke. Einführung in die Geometrie der Weben. - Basel: Verlags/Basel-Stuttgart, 1955.

[2]. V. Lazareva. Parallelizable webs formed by pencils of circles. - Webs and Quasigroups. - Kalinin, Kalinin State University, 1988, 74-77pp. (Rus).

[3]. On three-webs formed by pencils of circles. - Itogi nauki i tekhniki VINITI. - t.32(2005). - p.7-28 (Rus).