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Spectral analysis on Zeeman manifolds

Abstract: By a recent observation, the Laplace operator on the Riemannian manifolds this author used for isospectrality constructions is nothing but the Landau-Zeeman operator of finite many electrons orbiting in constant magnetic fields. In physics, this operator was used to explain Zeeman effect. The Riemann manifolds having this coincidence are called Zeeman manifolds. The most simple examples can be constructed on 2-step nilpotent Lie groups. This lecture proceeds with this case.

There is a natural representation, called Fock-Bargmann representation, of the complex Heisenberg Lie algebra on the Hilbert space of complex valued functions defined on Zeeman manifolds (the Hilbert norm is defined by a natural Gauss density). The physicists introduce the LZ-operator by means of this FB-representation and the Maxwell equations. The FB is a reducible representation. One of its irreducible subspaces is the Fock space spanned by the holomorphic polynomials. The projection onto the Fock space is an integral operator whose kernel was explicitly computed by Bargmann.

Although the FB-representation naturally acts on the total space of complex valued functions, it has been considered only on the Fock space and no thorough investigation of the extended representation is known in the literature. In this talk all the irreducible subspaces, called zones, with the corresponding projections will be explicitly determined. A thorough spectral analysis of these zones is the other objective of this talk. For instance, each zone is invariant under the action of the LZ-operator as well as flows such as the heat- resp. Schroedinger-flows. The corresponding zonal spectrum and zonal flows will also be explicitly determined. It is well known that the global objects define infinite physical quantities for the considered particles. In physics, these infinities are handled by the perturbative renormalization theory, which produces the desired finite quantities by differences of infinities. The most remarkable feature of this zonal theory is that all these quantities are finite ones on the zonal setting. This zonal spectral theory is a new non-perturbative tool by which the infinities appearing in QED can be handled.