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Some aspects of differential theories

`Abstract:` The talk is (in some sense) an outgrowth of the survey paper written together with *R. L. Lovas* under the same title and to be published in the forthcoming *Handbook of Global Analysis* edited by *D. Krupka* and *D. Saunders*. In the paper we focused on calculus on Frchet manifolds, based on Michal–Bastiani's concept of differentiability and emphasized that such tools may be useful also for differential geometers working in finite dimensions. In the talk I would like to discuss further aspects of differential theories, touched upon in the paper.

Since *Newton* it has been known that there is (or there must be) a strong and deep connection between physics ('natural philosophy'), geometry and calculus: a glimpse of their historical development makes clear the mutually enriching interaction between them. (Quite paradoxically, an adequate geometrical formulation of classical mechanics was achieved only in the 1950s.) Integrating physical content, geometric framework and appropriate calculus in a unified theory remains a permanent challenge and endeavour; it is enough to refer to the recent efforts of *N. Bourbaki*'s civilian relative, *Jet Nestruev*. Less ambitiously, in the talk I would like to sketch a unified theory for a more restricted area, which still covers a significant part of geometry, analysis and physics.

The story starts in the early 1920s, when an intensive study of the geometry of manifolds endowed with a 'system of paths' given by the Newtonian SODE

$$x^{i''} + 2G^i(x, x') = 0 \quad (i \in \{1, \dots, n\})$$

began. (It is assumed here that the functions G^i are positively homogeneous of degree 2 and smooth on the slit tangent bundle of the underlying manifold. For simplicity, I also suppose that the base manifold can be covered by a single coordinate system.) Based on this SODE, and using the tools of classical tensor calculus, a rich geometry, devided into the classes 'general', 'affine' and 'projective' geometry of paths was elaborated in the short period 1920-1930, due to the efforts of such eminent mathematicians as *L. Berwald*, *E. Cartan*, *J. Douglas*, *T. Y. Thomas*, *O. Veblen*, *H. Weyl* and others. The concept of a spray (Ambrose–Palais–Singer, 1960) and its near mutations, combined with an index-free construction of linear and non-linear connections made possible a reincarnation of path geometry in a modern framework. Having a spray, one can construct an Ehresmann connection in an intrinsic way, and from this, via linearization, a covariant derivative operator, called Berwald derivative. On the other hand, a Frlicher–Nijenhuis type theory of derivations of differential forms along the tangent bundle projection can be built up (theory of *E. Martínez*, *J. F. Cariñena* and *W. Sarlet*). These tools provide a perfectly adequate and efficient differential calculus for a systematic study of the geometry of paths. In the lecture I survey some problems and results concerning the different types of the metrizability of a spray; these metrizability problems are closely related to (in particular, coincide with) the inverse problem of calculus of variations.