Conference Book

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Toshiaki Adachi (Nagoya Institute of Technology, Nagoya, Japan)
Curvature logarithmic derivatives of curves and isometric immersions

Abstract: In my talk, we study isometric immersions by extrinsic shapes of some curves. In this area, Nomizu -Yano's theorem which characterizes extrinsic spheres is one of fundamental results. It says a submanifold M of a manifold $\widetilde{M}$ is an extrinsic sphere, which is a totally umbilic submanifold with parallel mean curvature vector, if and only if every circle on M is also a circle as a curve in $\widetilde{M}$. Here I give some extensions of this result.

Curvature logarithmic derivative of a smooth curve without inflection points is the ratio of the derivative of its first geodesic curvature to itself. I give a characterization of non-totally geodesic isotropic immersions in the sense of O'Neill by the amount of curvature logarithmic derivatives preserved by these immersions. Also I give a characterization of totally umbilic submanifolds with parallel normalized mean curvature vector in terms of curvature logarithmic derivatives of curves of order 2 preserved by immersions.

Ilka Agricola (Humboldt University Berlin, Berlin, Germany)
Eigenvalue estimates for generalized Dirac operators

Mukthar Abiodun Ahmed (Lagos State University, Lagos, Nigeria)

Alma Luisa Albujer (University of Murcia, Murcia, Spain)
Luis J. Alias
Global behaviour of maximal surfaces in Lorentzian product spaces

Abstract: A maximal surface in a 3-dimensional Lorentzian manifold is a spacelike surface with zero mean curvature. Here by *spacelike* we mean that the induced metric from the ambient Lorentzian metric is a Riemannian metric on the surface. The terminology *maximal* comes from the fact that these surfaces locally maximize area among all nearby surfaces having the same boundary. Besides of their mathematical interest, maximal surfaces and, more generally, spacelike surfaces with constant mean curvature are also important in General Relativity.

One of the most important global results about maximal surfaces is the Calabi-Bernstein theorem for maximal surfaces in the 3-dimensional Lorentz-Minkowski space $\mathbb{R}^3_1$, which, in parametric version, states that the only complete maximal surfaces in $\mathbb{R}^3_1$ are the spacelike planes. The

Calabi-Bernstein theorem in $\mathbb{R}^3_1$ can be seen also in a non-parametric form, and it establishes that the only entire maximal graphs in $\mathbb{R}^3_1$ are the spacelike planes; that is, the only entire solutions to the maximal surface equation

$$\mathrm{Div}\left(\frac{Du}{\sqrt{1-|Du|^2}}\right)=0, \quad |Du|^2<1.$$

on the Euclidean plane $\mathbb{R}^2$ are affine functions.

In this lecture we will introduce some recent results for maximal surfaces immersed into a Lorentzian product space of the form $M^2 \times \mathbb{R}$, where M^2 is a connected Riemannian surface and $M^2 \times \mathbb{R}$ is endowed with the Lorentzian metric $\langle,\rangle_M - dt^2$. In particular, we establish new Calabi-Bernstein results and study the parabolicity of such surfaces under certain appropriate hypothesis, deriving some interesting applications. These results are part of our joint work with Luis J. Alas and are contained in the two following references:

[1] A.L. Albujer and L.J. Alas, Calabi-Bernstein results for maximal surfaces in Lorentzian product spaces, preprint.
[2] A.L. Albujer and L.J. Alas, Parabolicity of maximal surfaces in Lorentzian product spaces, work in progress.

Dmitri Alekseevsky (Edinburgh University, Edinburgh, Great Britain)

Invariant bi-Lagrangian structures

Teresa Arias-Marco (University of Valencia, Valencia, Spain)
A. M. Naveira

The osculating rank of the Jacobi operator over g.o. spaces. A method for solving the Jacobi equation

`Abstract:` A Riemannian g.o. manifold is a homogeneous Riemannian manifold (M,g) on which every geodesic is an orbit of a one-parameter group of isometries. It is known that every simply connected Riemannian g.o. manifold of dimension ≤ 5 is naturally reductive. The first counter-example of a Riemannian g.o. manifold which is not naturally reductive is Kaplan's six-dimensional example. On the other hand, the second author and A. Tarrio have developed a method for solving the Jacobi equation in the manifold $Sp(2)/SU(2)$. This method is based on the fact that the Jacobi operator has constant osculating rank over naturally reductive spaces. Thus, the aim of this talk is to present the proof that the Jacobi operator also has constant osculating rank over g.o. spaces and, as a consequence, the Jacobi equation in the Kaplan example can be solve using the new method.

Andreas Arvanitoyeorgos (University of Patras, Rion, Greece)

Invariant Einstein metrics on some homogeneous spaces of classical Lie groups

`Abstract:` A Riemannian manifold (M,ρ) is called Einstein if the metric ρ satisfies the condition $\mathrm{Ric}(\rho)=c\cdot\rho$ for some constant c. Let G be a compact Lie group and H a closed subgroup so that G acts almost effectively on G/H. We investigate G-invariant metrics with additional symmetries, on some homogeneous spaces G/H of classical Lie groups. More precisely, let K be a closed subgroup of G with $H \subset K \subset G$, and suppose that $K = L' \times H'$, where $\{e_{L'}\} \times H' = H$. If we denote $L = L' \times \{e_{H'}\}$, then the group $\widetilde{G} = G \times L$ acts on G/H by $(a,b)\cdot gH = agb^{-1}H$, and the isotropy subgroup at eH is $\widetilde{H} = \{(a,b) : ab^{-1} \in H\}$.

We show that the set $\mathcal{M}^{\widetilde{G}}$ of $\widetilde{G}$-invariant metrics on $\widetilde{G}/\widetilde{H}$ is a subset of $\mathcal{M}^G$, the set of G-invariant metrics on G/H. Therefore, it is simpler to search for invariant Einstein metrics on $\mathcal{M}^{\widetilde{G}}$. In this way we obtain

existence results for Einstein metrics for certain quotients.

As a consequence, we obtain new invariant Einstein metrics on some Stiefel manifolds $SO(n)/SO(l)$, and on the symplectic analogues $Sp(n)/Sp(l)$. These metrics are in addition to the well known Einstein metrics obtained by G. Jensen in 1973. Furthermore, we show that for any positive integer p there exists a Stiefel manifold $SO(n)/SO(l)$ and a homogenous space $Sp(n)/Sp(l)$ which admit at least p $SO(n)$ (resp. $Sp(n)$)-invariant Einstein metrics.

Akira Asada (Sinsyu University, Takarazuka, Japan)
Zeta-regularized integral and Fourier expansion of functions on an infinite dimensional torus

`Abstract:` We have proposed a systematic way of application of zeta-regularization of a preasigned Schatten class operator to the calculus on a Hilbert space with the determinant bundle, which is named regularized calculus (Asada, A.: Regularized calculsu: An application of zeta regularization to infinite dimensional geoemtry and analysis, Int. J. Geom. Met. in Mod. Phys. 1(2004), 107-157, Asada, A.: Regularized volume form of the sphere of a Hilbert space with the determiant bundle, Proc. DGA2004, 397-409). In this talk, regularized calculus is applied to the Fourier expansion of functions on an infinite dimensional torus. Here the torus is the quotient of the Hilbert space with the determiannt bundle by the lattice in this space, not the qutient of the Hilbert space without the determiant bundle. It is also shown this torus have a de Rham type cohomology with the Poincare duality.

Sandor Bacso (University of Debrecen, Debrecen, Hungary)
Douglas spaces

Vladimir Balan (University Politehnica Bucharest, Bucharest, Romania)
Constant Mean Curvature submanifolds in (α,β)-Finsler spaces

`Abstract:` In 1998, based on the notion of Hausdorff measure, Z.Shen ([9]) introduced the notion of mean curvature on submanifolds of Finsler spaces. Recent advances in the theory of Finsler submanifolds of Busemann-Hausdorff type have been provided after 2002 by Z.Shen ([9]). Earlier rigorous attempts in this field, using functional analysis were developed by G.Bellettini and M. Paolini (e.g., [5, 6, 7]). Recently Z.Shen, and further M.Souza and K. Tenenblat ([10]) have studied submanifolds immersed in Finsler locally-Minkowski-Randers spaces from differential geometric point of view. In the present work, within the framework of (α,β) locally Minkowski Finsler spaces - which extend the Euclidean ones, we study the constant mean curvature field of submanifolds: the associated PDEs are explicitely detemined and for the 2-dimensional case, the Monge immersed surfaces are characterized and of the kind and the revolution minimal surfaces are completely described. The obtained results extend the known results obtained for Kropina and Randers spaces; relevant MAPLE plots exemplify the developed theory.

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Vitaly Balashchenko (Belarusian State University, Minsk, Belarus)
Eugene D. Rodionov, **Victor V. Slavskii**

Riemannian and Hermitian geometry of invariant structures on homogeneous manifolds

`Abstract:` We intend to present a brief observation of recent results on invariant affinor and (pseudo)Riemannian structures on homogeneous manifolds. Some basic notions, results and references can be found in [1-5].

Invariant *metric f-structures* ($f^3 + f = 0$, K.Yano) on naturally reductive homogeneous spaces were considered. We indicate the conditions under which these f-structures belong to the main classes of *generalized Hermitian geometry* such as *Kähler* (Kf), *Hermitian* (Hf), *Killing* ($Killf$), *nearly Kähler* (NKf), G_1f*-structures* (G_1f). As a particular case, it follows some results of V.F.Kirichenko, E.Abbena and S.Garbiero in Hermitian geometry. As is known, *canonical f-structures* on *homogeneous k-symmetric spaces* provide a remarkable vast class of invariant metric f-structures including almost Hermitian structures. First, canonical f-structures on homogeneous 4- and 5-symmetric spaces were completely characterized in the sense of generalized Hermitian geometry. Besides, partial results were also obtained for the cases $k = 4n, n \geq 1$ and some others. Moreover, recently four canonical f-structures on homogeneous 6-symmetric spaces were also completely studied in this respect. Many particular examples with respect to families of invariant (pseudo)Riemannian metrics were investigated in detail. They are the flag manifolds $SU(3)/T_{max}$, $Sp(3)/Sp(1)\times Sp(1)\times Sp(1)$, $SO(n)/SO(2)\times SO(n-3),\ n \geq 4$, the Stiefel manifold $SO(4)/SO(2)$ and some others. Specifically, we present invariant Killing f-structures with non-naturally reductive metrics as well as construct invariant Kähler f-structures on some naturally reductive not locally symmetric homogeneous spaces.

The classification of three-dimensional Lie groups admitting a left-invariant Lorentzian metric and an almost harmonic Schouten-Weyl tensor was obtained. We introduce a skew-symmetric 2-tensor using the contraction of the Schouten-Weyl tensor in the direction of an arbitrary vector. The structure of three-dimensional Lie groups and Lie algebras endowed with a left-invariant Riemannian metric and a harmonic skew-symmetric 2-tensor above mentioned was investigated. As a result, the

complete classification of such Lie groups and Lie algebras in directions of harmonic vectors was obtained. Besides, three-dimensional metric Einstein algebras for three-dimensional Lie groups with left-invariant Lorentzian metrics were also found.

Some formulas for calculating the Schouten-Weyl tensor and the Weyl tensor in terms of the structural constants of the metric triple Lie algebras of a homogeneous Riemannian space were obtained. Almost harmonic and harmonic homogeneous Riemannian metrics on some particular classes of homogeneous spaces were found. Invariant metrics with the harmonic Weyl tensor on the generalized Berger-Wallach spaces were also investigated.

Finally, theorems of general character on invariant pseudo-Riemannian metrics constructed by means of canonical invariant structures on generalized symmetric spaces were proved. We illustrate here a fundamental role of the canonical almost product and almost complex structures on homogeneous k-symmetric spaces.

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Cornelia - Livia Bejan (University "Al.I.Cuza", Iasi, Romania)

Generalizations of Einstein manifolds

Abstract: For manifolds carrying some structures, by taking particular expressions of the energy-momentum tensor in the field equations for the interaction of gravitation with other fields, one obtains some special manifolds, as for instance: Einstein, quasi-Einstein, η-Einstein, Kahler-Einstein, Kahler η-Einstein, etc. We deal here with locally decomposable Riemannian manifolds, on which the energy-momentum tensor takes certain expressions. We construct some examples, study some curvature properties and generalize a result of K. Yano and M. Kon.

Ian Benn (University of Newcastle, Newcastle, NSW, Australia)

Differential forms relating twistors to Dirac fields

Abstract: In a conformally flat four-dimensional space one can use a twistor to 'raise the helicity' of a scalar field that satisfies the conformally-covariant wave equation, to produce a solution to the massless Dirac equation. One can alternatively regard this as using a scalar field to construct a first-order operator that maps solutions to the twistor equation to solutions to the massless Dirac equation. This suggests a generalisation to all conformally flat spaces which proves fruitful.

I shall show how, in a conformally flat space of arbitrary dimension and signature, one can express the most general first-order operator taking twistors to massless Dirac solutions. Such operators involve forms of various degrees satisfying an interesting conformally-covariant equation. Solutions to this equation include those to the scalar wave equation and the (generalised) Maxwell equations. First-order symmetry operators for this equation will be constructed from those for the twistor and Dirac equations. Such symmetry operators involve conformal Killing-Yano tensors, and change the degree of the forms.

Behroz Bidabad (Amirkabir university of Technology, Tehran, Iran)
A. Asanjarani
A classification of complete Finsler manifolds of constant curvature

`Abstract:` Here, in continuation of our previous work in DGA journal, an special case of a certain second order differential equation is studied and a classification of all complete Finsler manifolds of constant flag curvature is obtained.

Richard Bishop (University of Illinois at Urbana-Champaign, USA)

Adara-Monica Blaga (West University from Timisoara, Romania)

Albert Borbely (Kuwait University, Safat, Kuwait)
Unclouding the sky of negatively curved manifolds

`Abstract:` The problem of finding geodesics that avoid certain obstacles in negatively curved manifolds has been studied in different situations. It found applications in several areas like the existence of bounded geodesics in manifolds with finite volume and the construction of proper closed invariant subsets of the unit tangent tangent bundle with large footprints. In this talk we review some of the applications and give a generalization of the following Unclouding Theorem of J. Parkkonen and F. Paulin: there is a constant $s_0 = 1.534$ such that for any Hadamard manifold M with curvature ≤ -1 and for any family of disjoint balls or horoballs $\{C_a\}_{a\in A}$ and for any point $p \in M - \cup_{a\in A} C_a$ if we shrink these balls uniformly by s_0 one can always find a geodesic ray emanating from p that avoids the shrunk balls. It will be shown that in the theorem above one can replace the balls by arbitrary convex sets.

Wlodzimierz Borgiel (Warsaw University of Technology, Warsaw, Poland)
The Gravitational Field of the Robertson-Walker Spacetime

Alexander A. Borisenko (Kharkov National University, Kharkov, Ukraine)

Jan Brajercik (University of Presov, Presov, Slovakia)

Miguel Brozos-Vazquez (University of Santiago de Compostela, Santiago de Compostela, Spain)
Eduardo Garcia-Rio (University of Santiago de Compostela, Santiago de Compostela, Spain)
Peter Gilkey (University of Oregon, Eugene, OR, USA)
The complex Jacobi operator and the complex Osserman condition

Abstract: Let (M, g) be a Riemannian manifold. The Osserman condition is given by the constancy of the eigenvalues of the Jacobi operator. More specifically a manifold is said to be pointwise Osserman if the eigenvalues of $\mathcal{J}(x) := R(x, \cdot)x$ do not depend on the unit x at each point of M. This condition has been used to characterize 2-point-homogeneous manifolds. Let (M, g, J) be an almost-Hermitian manifold; we study a complex version of the Osserman condition. The complex Jacobi operator is defined on (M, g, J) as $\mathcal{J}(\pi_x) = \mathcal{J}(x) + \mathcal{J}(Jx)$ for a complex line $\pi_x := span\{x, Jx\}$. First we identify the setting of algebraic curvature tensors where the complex Jacobi operator determines the curvature tensor. Then some general results concerning complex Osserman manifolds are established, including a complete description of the eigenvalue structure of the complex Jacobi operator. The final result is the complete classification of complex Osserman Kähler manifolds in dimension 4.

Esteban Calvino-Louzao (University of Santiago de Compostela, Santiago de Compostela, Spain)

Curvature properties of Riemann extensions

Abstract: The cotangent bundle of any affine manifold can be naturally equipped with a pseudo-Riemannian metric of neutral signature: the Riemann extension of the torsion free connection. Riemann extensions are a powerful tool to relate properties from affine and pseudo-Riemann geometry. A remarkable fact is that any four-dimensional Riemann extension is self-dual and they provide the underlying geometry of hypersymplectic structures.

We investigate curvature properties of Riemann extensions by using curvature operators defined by the Riemann curvature tensor. Main attention will be paid to the spectral properties of the Jacobi operator and the skew-symmetric curvature operator.

Jose F. Carinena (Unversity of Zaragoza, Zaragoza, Spain)

Integrability of Lie systems and some of its applications in physics

Marco Castrillon Lopez (University of Santiago de Compostela, Santiago de Compostela, Spain)

J. Munoz Masque (CSIC, Madrid, Spain)

Lie algebra pairing and the Lagrangian and Hamiltonian equations in gauge-invariant problems

Abstract: Let $P \to M$ be a principal G-bundle over a manifold M endowed with a pseudo-Riemannian metric g. The metric g and the Cartan-Killing metric in the Lie algebra $\mathfrak{g}$ define the Yang-Mills Lagrangian in the well-known way. Assume that G is semisimple so that the Killing metric is nondegenerate. We prove that the Euler-Lagrange equations of the Lagrangian obtained by substituting the Killing metric by any other adjoint-invariant nondegenerate symmetric bilinear form in $\mathfrak{g}$ are the same as the original Yang-Mills equations. A similar result is obtained for the corresponding Hamiltonian equations. The exposition also shows the structure of these other bilinear forms.

On the other hand, it is shown that the Hamiltonian equations of any gauge invariant problem are essentially the same as their Euler-Lagrange equations if some simple regularity assumptions are made on the Lagrangian. Actually, the set of solutions to the Hamiltonian equations trivially fibers over the set of extremals of the variational problem. The

tangent structure (Jacobi fields) is also analyzed.

Hana Chuda (Tomas Bata University, Zlin, Czech Republic)
Josef Mikes (Palacky University, Olomouc, Czech Republic)
$\mathbb{F}_2$-planar mappings with certain initial conditions

`Abstract:` In this paper we investigate special $\mathbb{F}_2$-planar mappings of n-dimensionaly (pseudo-) Riemannian spaces $V_n \to \bar{V}_n$ with metric satisfying at a finite number of points following conditions $\bar{g}(X,Y) = f\,g(X,Y)$.

It comes out that even under this conditions it holds that $\mathbb{F}_2$-planar mapping is homothetic.

Balazs Csikos (Eotvos Lorand University, Budapest, Hungary)
Extendability of the Kneser-Poulsen Conjecture to Riemannian Manifolds

`Abstract:` The Kneser-Poulsen conjecture (still open in dimensions ≥ 3) claims that if $P_1,\ldots,P_N$ and $Q_1,\ldots,Q_N$ are points in the Euclidean space, such that $d(P_i,P_j) \geq d(Q_i,Q_j)$ for all $1 \leq i \leq j \leq N$, and $r_1,\ldots,r_N$ are given positive numbers, then the volume of the union of the balls $B(P_i,r_i)$, $i=1,\ldots,N$ is not less than the volume of the union $\bigcup_{i=1}^{N} B(Q_i,r_i)$.

In this talk, we study the question what class of Riemannian manifolds the conjecture can be extended to. First we overview some volume variation formulae, in particular a Schläfli-type formula in Einstein manifolds, which enable us to prove special cases of the conjecture not only in the Euclidean, but also in the hyperbolic and spherical spaces. These results give hope that the conjecture will turn out to be true in these spaces as well. Secondly, we prove that if M is a complete connected Riemannian manifold, in which the Kneser-Poulsen conjecture holds, then M is a simply connected space of constant curvature.

Lenka Czudkova (Masaryk University, Brno, Czech Repubic)
Jana Musilova (Masaryk University, Brno, Czech Repubic)
Non-holonomic constraint forces in theory of field (geometry & physics)

`Abstract:` We study non-holonomic constraint forces using the geometrical theory of non-holonomic constrained systems on fibered manifolds with n-dimensional base and their prolongations (proposed by Krupkova). Properties of so-called Chetaev forces, that arise as a result of this geometrical theory, are illustrated on concrete physical examples concerning continuum mechanics and elasticity theory. By means of these examples the geometrical as well as physical aspects of the theory are analyzed. We sketch the quite general approach to a problem of non-holonomic constraint forces.

Pantelis Damianou (University of Cyprus, Cyprus)
Herve Sabourin, **Pol Vanhaecke**
Transverse Poisson structures to coadjoint orbits in simple Lie algebras

`Abstract:` We study the transverse Poisson structure to adjoint orbits in a complex semi–simple Lie algebra. The problem is first reduced to the case of nilpotent orbits. We prove then that in suitably chosen quasi–homogeneous coordinates the quasi–degree of the transverse Poisson structure is -2. In the particular case of subregular nilpotent orbits we show that the structure may be computed by means of a simple

determinant formula, involving the restriction of the Chevalley invariants on the slice. In addition, using results of Brieskorn and Slodowy, the Poisson structure is reduced to a three dimensional Poisson bracket, intimately related to the simple rational singularity that corresponds to the subregular orbit.

Manuel de Leon (CSIC, Madrid, Spain)
Functionally graded materials

`Abstract:` A functionally graded material has conjugate symmetry groups but it is not uniform. Nevetherles it could enjoy some homogeneity properties that can be characterized in terms of the integrability of some G-structures.

Cemile Elvan Dinc (Kadir Has University, Istanbul, Turkey)
Sezgin Altay, **Fusun Ozen**
Kahler spaces with recurrent H-projective curvature tensor

`Abstract:` A Kahler space K_n is a Riemannian space in which, along with the metric tensor $g_{ij}(x)$, and affine structure $F_i^h(x)$ is defined that satisfies the relations

$$F_\alpha^h F_i^\alpha = e\delta_i^h \; ; \quad F_{(i}^\alpha g_{j)\alpha} = 0 \; ; \; F_{i,j}^h = 0, \quad where \quad e = \pm 1, 0.$$

Here and in what follows ″,″ denotes a covariant derivative.

If $e = -1$, then K_n is an elliptically Kahler space K_n^-, if $e = +1$, then K_n is a hyperbolically Kahler space K_n^+, and if $e = 0$ and $R_g \| F_i^h \| = m \leq \frac{n}{2}$, then K_n is an m-parabolically Kahler space $K_n^{0(m)}$. Necessarily, the spaces K_n^+, K_n^- and K_n^0 are of an even dimension.

The spaces K_n^- were first considered by P. A. Shirokov in 1966.

In this paper, we consider the holomorphically projective curvature tensor in the space K_n^-. Problems containing in this study are as follows:

Every H- projective recurrent Kahler space is recurrent. It' s shown that the curvature tensor of H- projective recurrent Kahler- Einstein space is harmonic and if this space with constant curvature has positive definite metric then it's flat.

The notion of quasi- Einstein space was introduced by M. C. Chaki and R. K. Maity in 2000. According to him a non- flat Riemannian space $(n > 2)$ is said to be a quasi- Einstein space if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies the condition

$$S(X, Y) = ag(X, Y) + bA(X)A(Y),$$

where a and b are scalars of which $b \neq 0$. A is a nonzero 1- form such that $g(X, \rho) = A(X)$ for all vector fields X and ρ is an unit vector field. We searched relations between the quasi- Einstein spaces and H- projective recurrent Kahler spaces; and then noticed some relations between these two spaces supporting a theorem.

Subsequently, we obtained a relation between the Ricci tensor and the curvature tensor by defining a special vector field $u_i = \lambda_s F_i^s$ in a positive definite Kahler space with non- constant scalar curvature and also it is shown that the ricci tensor' s rank is two and its eigenvalues are zero and $\dfrac{R}{2}$.

On the other hand, we proved that the length of the special vectors defined by the recurrence vector λ_l are constant in a H- projective Kahler space with non- zero scalar curvature.

In the last section, we presented an example of H- projective recurrent Kahler space. For this, H- projective recurrent Kahler space is taken as

the subset $M \subset R^4$ where M is an open connected subset of R^4 and (x^1, x^2, x^3, x^4) are the Cartesian coordinates in R^4. For the metric tensor g_{ij} and the tensor field F_j^h given in our subsequent work, using the recurrency condition $P_{hijk,l} = \lambda_l P_{hijk}$, we have a linear first order partial differential equation. By solving the Lagrange system, the general solution of $\varphi(x^1, x^3)$ is found where the function φ is one of the components of the metric tensor of H- projective recurrent Kahler space. In this case, the line element of this space is determined. We also want to mention that the open connected subset $M \subset R^4$ admits a almost L structure.

AMS Classification: 53B20, 53B15

Key Words : Kahler manifold, recurrent space, special quasi Einstein manifold, Ricci tensor, Holomorphically projective curvature tensor.

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Miroslav Doupovec (Brno University of Technology, Brno, Czech Republic)
Wlodzimierz M. Mikulski (Jagiellonian University, Krakow, Poland)
Extension of connections

`Abstract:` Denote by J^rY or $\overline{J}^rY$ or $\widetilde{J}^rY$ the r-th order holonomic or semiholonomic or nonholonomic jet prolongation of a fibered manifold $Y \to M$, respectively. Given a projectable classical linear connection on Y, we generalize the well known Ehresmann prolongation of a connection $\Gamma : Y \to J^1Y$ in the following way: First we introduce an extension of Γ into an r-th order holonomic connection $Y \to J^rY$. Then we introduce an extension of r-th order holonomic connections into s-th order ones, $s > r$. We also describe holonomizations $\overline{J}^rY \to J^rY$ and $\widetilde{J}^rY \to J^rY$ of semiholonomic and nonholonomic jets, respectively. Finally we completely classify some of above geometric constructions.

Ugur Dursun (Istanbul Technical University, Istanbul, Turkey)
On Minimal Hypersurfaces of Hyperbolic Space H^4 with Zero Gauss-Kronecker Curvature

`Abstract:` We determine minimal hypersurfaces of the hyperbolic space $H^4(-1)$ with identically zero Gauss-Kronecker curvature. Such a hypersurface is the image of a subbundle spanned by a timelike vector field of the normal bundle of a totally geodesic surface of the de Sitter space $S^4_1(1)$ under the normal exponential map. We also construct some examples.

Zdenek Dusek (Palacky University, Olomouc, Czech Republic)
Homogeneous geodesics on Riemannian and pseudo-Riemannian manifolds

`Abstract:` Let M be a homogeneous Riemannian manifold (homogeneous space). A geodesic $\gamma(t)$ on M is homogeneous if it is an orbit of a one-parameter group of isometries. If all geodesics of M are homogeneous, M is called a g.o. manifold.

It is well known that all naturally reductive spaces are g.o. manifolds. G.o. manifolds which are not naturally reductive are characterized by the degree of the geodesic graph (which is a nonlinear map in this case) and the degree of the g.o. manifold. For the simplest examples, which are in dimensions 6 and 7, the degree of these g.o. manifolds is equal to 2. The first example of a g.o. manifold of higher degree (namely, of degree 3) is the 13-dimensional generalized Heisenberg group. Recently, a g.o. manifold of degree 4 was explicitly described and it will be presented in the talk. It is the Riemannian flag manifold $\mathrm{SO}(7)/\mathrm{U}(3)$.

On pseudo-Riemannian manifolds, homogeous geodesics have new interesting features. First, for null homogeneous geodesic, the parameter of the orbit may not be the affine parameter of the geodesic. Second, there are examples of homogeneous spaces, for which the initial vectors of homogeneous geodesics through the origin fill in the open dense subset in the tangent space, but not all of it. These spaces are called almost g.o. spaces. In the talk, examples will be shown and conjectures in this direction will be presented.

Maria Dyachkova (Kazan State University, Kazan, Russia)
On Hopf bundle analogue for semiquaternion algebra

`Abstract:` The subject of the talk is to study the principle algebra

bundle by it's subalgebra. Let $\mathfrak{A}$ be a 4-dimensional associative unital nonreducible semiquaternion algebra [1]. The basic units multiply by the following rules: $e_1^2 = -1, \ e_1e_2 = -e_2e_1 = e_3, \ e_1e_3 = -e_3e_1 = -e_2, \ e_2^2 = e_3^2 = e_2e_3 = e_3e_2 = 0$. Consider the degenerate scalar product:$(x, y) = x_0y_0 + x_1y_1$. Thus the algebra $\mathfrak{A}$ has the 4-dimensional semieuclidean space structure ${}_2\mathbb{R}^4$ with rank 2 semimetric.

The set G consisting of invertible elements of $\mathfrak{A}$ is a Lie group, which is connected but not simply connected. This algebra contains 2-dimensional subalgebras isomorphic either to complex or dual number algebra. Let H be the set of invertible elements of dual algebra with the basis $\{1, e_2\}$. H is Lie subgroup of G. A semiquaternion can be reprezented in the following way: $x = z_1 + e_1z_2$, here $z_1 = x_0 + x_2e_2$, $z_2 = x_1 + x_3e_2$.

The bundle $(G, \pi, G/H)$ of right cosets is a principle locally trivial bundle with the structure group H. The base is the subset $M = \{[z_1 : z_2] \in P(e_2) | |x|^2 \neq 0\}$ of projective line $P(e_2)$ over the dual algebra without a point. The canonical projection is $\pi(x) = (\overline{z}_2 : z_1)$.

The set of invertible elements with the module one (the *semieuclidian sphere* ${}_2S^3 = \{x \in \mathfrak{A} | |x|^2 = 1\}$) is the 2-fold cover of the special orthogonal group $SO(3)$. It is an analogue of the Hopf bundle.

Consider the restriction of the bundle $(G, \pi, G/H)$ to the irreducible unit sphere ${}_2S^3$, i.e. the bundle $\pi :_2 S^3 \to M$. Then the restriction of the group H to ${}_2S^3$ is the Lie group S. The bundle $({}_2S^3, \pi, M)$ is principle locally trivial with total space and fiber being Lie groups ${}_2S^3$ and S, respectively .

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Askar Dzhumadildaev (Institute of Mathematics, Almaty, Kazakhstan)
New tensor operations on vector fields generalising Lie commutator

Juergen Eichhorn (Greifswald University, Greifswald, Germany)
The Ricci flow on open manifolds

Alberto Enciso (Universidad Complutense de Madrid, Madrid, Spain)
Daniel Peralta-Salas (Universidad Carlos III de Madrid, Madrid, Spain)
A dynamical systems approach to Green functions on Riemannian manifolds

`Abstract:` In this talk we will qualitatively analyze some topological and geometrical properties of the Green functions of the Laplace-Beltrami operator on an open Riemannian manifold. Our approach relies on a suitable combination of techniques from dynamical systems, analytic geometry and harmonic analysis. Particular emphasis shall be made on the characterization and study of the critical points of the Green functions. Physically, this problem naturally arises when extending the classical Electrostatic theory to manifolds.

Alexander A. Ermolitski (Belarusian State Pedagogical University, Minsk, Belarus)
On Poincare conjecture

`Abstract:` Let M^n be a compact, closed, smooth manifold. Then there

exist a Riemannian metric g on M^n and a smooth triangulation of M^n. Both the structures make possible to prove the following

Theorem 1. *The manifold M^n has a decomposition $M^n = C^n \cup K^{n-1}$, $C^n \cap K^{n-1} = \emptyset$, where C^n is a n-dimensional cell and K^{n-1} is a connected union of some $(n-1)$-simplexes of the triangulation.*

K^{n-1} is called a *spine* of M^n.

Using the theorem 1 we have obtained

Theorem 2 (Poincare). *Let M^3 be a compact, closed, smooth, simply connected manifold of dimension 3. Then M^3 is diffeomorphic to S^3, where S^3 is the sphere of dimension 3.*

Proof of the theorem 2 consists of the following steps.

Proposition 3. *If δ^2 is a 2-simplex with boundary in a spine K^2 of the manifold M^3 then δ^2 can be retracted by means of a homeomorphism.*

Proposition 4. *Any spine K^2 of the manifold M^3 can be rebuilt in such way that a new spine $\bar{K}^2$ has a 2-simplex with boundary.*

It follows from propositions above that the manifold M^3 has a spine K^1 consisting of 1-simplexes and K^1 is a tree.

Proposition 5. *The tree K^1 can be retracted to a point p by means homeomorphisms.*

Thus, the manifold M^3 is a union of a cell C^3 and a point p and it follows that M^3 is homeomorphic and diffeomorphic to S^3.

Jost-Hinrich, Eschenburg (Universitat Augsburg, Augsburg, Germany)

Riemannian Geometry and Submanifolds

Younhei Euh (Sungkyunkwan University, Suwon, Korea)
Jeong Hyeong Park (Sungkyunkwan University, Suwon, Korea)

K. Sekigawa

Tricerri-Vanhecke Bochner flat almost Hermitian surfaces

`Abstract:` Our recent work on the almost Hermitian surfaces will be summarized. We study and characterize curvature properties of four-dimensional almost Hermitian manifolds with vanishing Bochner curvature tensor as defined by Tricerri and Vanhecke. We also find out several examples of the obtained results.

Vasyl Fedorchuk (Pedagogical Academy, Krakow, Poland; National Ukrainian Academy of Sciences, Lviv, Ukraine)

On first-order differential invariants of the non-conjugate subgroups of the Poincare group $P(1,4)$

`Abstract:` The number of the non-equivalent functional bases of the first-order differential invariants for all non-conjugate subgroups of the Poincaré group $P(1,4)$ is established. All these bases have been classified according to their dimensions. The application of the results obtained to the construction of classes of the first-order differential equations in the space $M(1,4) \times R(u)$ invariant under these subgroups are considered. Among these classes there are ones invariant under following subgroups of the group $P(1,4)$: $O(1,4)$, $P(1,3)$, $\widetilde{G}(1,3)$, $O(4)$, $E(4)$ and etc.

Agota Figula (Debrecen University, Debrecen, Hungary)

Multiplications on symmetric and reductive spaces

`Abstract:` The symmetric spaces and the reductive spaces are im-

potant classes of homogeneous spaces G/H. These spaces are defined by conditions on the triples $(\mathfrak{g}, \mathfrak{h}, \mathfrak{m})$, where $\mathfrak{g}$ and $\mathfrak{h}$ are Lie algebras of the Lie groups G and H, respectively, and $\mathfrak{m}$ is a complement to $\mathfrak{h}$ in $\mathfrak{g}$.

In this talk I would like to classify global multiplications on the spaces G/H, where all group axioms hold only the associative law is dismissed. These multiplications correspond in the case of symmetric spaces to differentiable Bol-loops, in the case of reductive spaces to differentiable links A-loops.

The exponential image exp $\mathfrak{m}$ contains such elements of G which are image of a global differentiable sharply transitive section $\sigma : G/H \to G$. This fact is fundamental to the classification of differentiable loops L. Namely, we can consider the Lie group G topologically generated by the left translations of L and after this we determine to the homogeneous spaces G/H all sections, on which L is defined.

Thomas Friedrich (Humboldt University Berlin, Berlin, Germany)
G_2-manifolds with parallel characteristic torsion

Pedro Luis Garcia (University of Salamanca, Salamanca, Spain)
Cesar Rodrigo (Academia Militar Lisboa, Lisboa, Portugal)
Hamiltonian approach to second variation for constrained problems

`Abstract:` A key concept in the geometric theory of the calculus of variations is that of Poincaré-Cartan form. In the presence of differential constraints defined by zeros of some differential operator $\Phi\colon \Gamma(X,Y) \to \Gamma(X,E)$, and under certain conditions of regularity of Φ, we ensure the existence of a new differential operator $N_s\colon \Gamma(X,E) \to \Gamma(X,VY)$ that allows to construct a "constrained" Poincaré-Cartan form $\widetilde{\Theta}_L$ associated to any lagrangian density L and Φ. This new object satisfies the usual Lepage congruences in this constrained framework.

The approach presented allows to characterize critical sections, second variation and the Hessian metric and quadratic form at critical sections for the constrained problem, in terms of $\widetilde{\Theta}_L$, without need of considering the free variational problem arising from the Lagrange multiplier rule. We study the usual notions for second variation: Hessian metric, conditions for maxima/minima (stability), Jacobi differential operator, and Jacobi vector fields, stressing the main differences and misconceptions that are likely to arise in the Lagrange multipliers approach.

This approach is illustrated by a nice example from the theory of constant mean curvature surfaces.

Peter Gilkey (University of Oregon, Eugene, OR, USA)
Miguel Brozos-Vazquez (University of Santiago de Compostela, Santiago de Compostela, Spain)
Eduardo Garcia-Rio (University of Santiago de Compostela, Santiago de Compostela, Spain)
S. Nikcevic (SANU, Belgrade, Serbia)
R. Vazquez-Lorenzo
Stanilov-Tsankov-Videv Theory

`Abstract:` Let R be the curvature operator, let J be the Jacobi operator, and let p be the Ricci operator of a pseudo-Riemannian manifold M. One studies commutativity properties of these operators - for example M is said to be Jacobi-Videv if $J(x)p = pJ(x)$ for all x and skew-Tsankov if $R(x,y)R(z,w) = R(z,w)R(x,y)$ for all x,y,z,w. We examine how these purely algebraic properties are related to the underlying geometry of

the manifold and we present a number of examples are presented in the context of Walker geometry.

Simon Gindikin (Rutgers University, Piscataway, USA)
Dual objects for symmetric spaces or back to Pluecker

Nikolaj Glazunov (National Aviation University, Kiev, Ukraine)
Differential Geometry and Justification of Conjectures of Number Theory and Mathematical Physics

`Abstract:` We construct, investigate and apply the methods of differential geometry and homological/homotopical algebra to justification of conjectures of number theory and mathematical physics.

Important ingredients of our methods is the globalization of the problem and developing and applying methods in the framework of the globalization.

Earlier D. Krupka and D. Krupka with colleagues have developed and applied methods of globalization to some problems of mathematics and mathematical physics and obtained profound results.

We will consider problems from geometry of numbers and mirror symmetry in string theory and justification of conjectures about possible solutions of the problems.

Vladislav V. Goldberg (New Jersey Institute of Technology, New Jersey, USA)
M. A. Akivis
Local algebras of a differentiable quasigroup

`Abstract:` We study local differentiable quasigroups and their local algebras defined by the second and third orders terms of Taylor's decomposition of a function defining an operation in local loops. In local algebras, we define the commutators and associators connected by a third-degree relation generalizing the Jacobi identity in Lie algebras. Hofmann and Strambach named the local algebras mentioned above Akivis algebras.

In general, an Akivis algebra does not uniquely determine a differentiable quasigroup, but for Moufang and Bol quasigroup, it enjoys this property.

We consider also prolonged Akivis algebras and prove that a local algebra defined in a fourth-order neighborhood uniquely determines a monoassociative quasigroup.

The last two results give a generalization of the classical converse third Lie theorem on determination of a local Lie group by its Lie algebra.

As an illustration, we consider local differentiable quasigroups defined on the Grassmannian $\mathbb{G}(1, r+1)$ by a triple of hypersurfaces in the projective space $\mathbb{P}^{r+1}$.

Hubert Gollek (Humboldt University at Berlin, Berlin, Germany)
Projective duals of curves in Riemannian manifolds

Midori Goto (Fukuoka Institute of Technology, Fukuoka, Japan)
Generalized Liouville manifolds

`Abstract:` I will report the following results of joint works with Kunio Sugahara:

(1)The study of the Liouville structure was initiated by C.G.F.Jacobi in

1866. We generalize the Liouville structure for indefinite/definite metrics and define the generalized Liouville structure, which include the case of the Riemannian metric. One of our main results is a classifiction of 2-dimensional Lorentz-Liouville manifolds.

(2)We obtain a related result as above on semi-symmetric 3-spaces.

Galina Guzhvina (Westfaelische Wilhelms Universitaet, Muenster, Germany)

The Action of the Ricci Flow on Almost Flat Manifolds

`Abstract:` A compact Riemannian manifold M^n is called ε-flat if its curvature is bounded in terms of the diameter as follows:

$$|K| \leq \varepsilon \cdot d(M)^{-2},$$

where K denotes the sectional curvature and $d(M)$ the diameter of M: If one scales an ε-flat metric it remains ε-flat.

By almost flat we mean that the manifold carries ε-flat metrics for arbitrary $\varepsilon > 0$.

The (unnormalized) Ricci flow is the geometric evolution equation in which one starts with a smooth Riemannian manifold (M^n, g_0) and evolves its metric by the equation:

$$\frac{\partial}{\partial t} g = -2ric_g, \tag{1}$$

where ric_g denotes the Ricci tensor of the metric g.

When M^n is compact, one often considers the normalized Ricci flow

$$\frac{\partial}{\partial t} g = -2ric_g + \frac{2sc(g)}{n} g, \tag{2}$$

where $sc(g)$ is the average of the scalar curvature of M^n. Under the normalized flow the volume of the solution metric is constant in time, equations (1) and (2) differ only by a change of scale in space by a function of t and change of parametrization in time (see [?]).

The subject of my thesis was study how the Ricci flow acts on almost flat manifolds. We show that on a sufficiently flat Riemannian manifold (M, g_0) the Ricci flow exists for all $t \in [0, \infty)$, $\lim_{t\to\infty} |K|_{g(t)} \cdot d(M, g(t))^2 = 0$ as $g(t)$ evolves along (1), moreover, if $\pi_1(M, g_0)$ is abelian, $g(t)$ converges along the Ricci flow to a flat metric. More precisely, we establish the following result:

Main Theorem (Ricci Flow on Almost Flat Manifolds.)

In any dimension n there exists an $\varepsilon(n) > 0$ such that for any $\varepsilon \leq \varepsilon(n)$ an ε-flat Riemannian manifold (M^n, g) has the following properties:

(i) the solution $g(t)$ to the Ricci flow (1)

$$\frac{\partial g}{\partial t} = -2ric_g, \quad g(0) = g,$$

exists for all $t \in [0, \infty)$,

(ii) along the flow (1) one has

$$\lim_{t\to\infty} |K|_{g_t} \cdot d^2(M, g_t) = 0$$

(iii) $g(t)$ converges to a flat metric along the flow (1), if and only if the fundamental group of M is (almost) abelian (= abelian up to a subgroup of finite index).

Izumi Hasegawa (Hokkaido University of Education, Japan)
Kazunari Yamauchi

Conformally-projectively flat statistical structures on tangent bundles over statistical manifolds

`Abstract:` Let (M, h, ∇) be a statistical manifold. Semi-Riemannian manifold with Levi-Civita connection is a typical example of statistical manifold. In Riemannian geometry we have the following well-known

Fact A. *Let (M, h) be a semi-Riemannian manifold of dimension $n(\geq 2)$. Then the tangent bundle TM over M with complete lift metric h^C is conformally flat if and only if (M, h) is of constant curvature.*

There are many statistical structures on the tangent bundle TM over the statistical manifold (M, h, ∇). We want to investigate the statistical versions of the above theorem. For example, the pair (h^C, ∇^C) of complete lift of h and complete lift ∇ is a statistical structure on TM. In this case we have the following

Theorem 1. *Let (M, h, ∇) be a statistical manifold of dimension $n(\geq 2)$. Then (TM, h^C, ∇^C) is conformally-projectively flat if and only if (M, h, ∇) is of constant curvature. Especially if (TM, h^C, ∇^C) is ± 1-conformally flat, then (M, h, ∇) and (TM, h^C, ∇^C) are locally flat.*

In this conference we talk about the conformal-projective flatness of some other statistical structures on TM over (M, h, ∇).

Irena Hinterleitner (FSI VUT Brno, Czech Republic)
On conformally-projective mappings

Jaroslav Hrdina (Masaryk University, Brno, Czech Republic)

Cristina-Elena Hretcanu (Stefan cel Mare University, Suceava, Romania)

Examples of induced structures on submanifolds in Riemannian manifolds with special structures

`Abstract:` The purpose of this paper is to give an effective construction for induced structures on product of spheres of codimension r in an Euclidean space (of dimension m, $m > r$) endowed with an $(a, \varepsilon)f$ Riemannian structure or a golden structure.

First of all, we consider a m-dimensional Riemannian manifold $\widetilde{M}$ endowed with a pair $(\widetilde{P}, \widetilde{g})$, where $\widetilde{g}$ is a Riemannian metric and $\widetilde{P}$ is an (1,1) tensor field such that $\widetilde{P}^2 = \varepsilon Id$, $\varepsilon \in \{1, -1\}$ (where Id is the identity on $\widetilde{M}$) and it satisfies $\widetilde{g}(\widetilde{P}U, \widetilde{P}V) = \widetilde{g}(U, V)$ for every tangent vector fields $U, V \in \chi(\widetilde{M})$). The pair $(\widetilde{P}, \widetilde{g})$ induces on any submanifold M of codimension r in $\widetilde{M}$ $(r < m)$, a structure denoted by $(P, g, \xi_\alpha, u_\alpha, a_{\alpha\beta})$, where P is an (1,1)-tensor field on M, ξ_α are tangent vector fields on M, u_α are 1-forms on M and $a := (a_{\alpha\beta})_r$ is a $r \times r$ matrix where its entries are real functions on M $(\alpha, \beta \in \{1, ..., r\})$. These structures generalize the almost r-contact and r-paracontact structures. We called this kind of structure an $(a, \varepsilon)f$ Riemannian structure.

Secondly, we define a golden structure on a Riemannian manifolds $(\widetilde{M}, \widetilde{g})$, determined by an (1,1)-tensor field $\widetilde{P}$ on $\widetilde{M}$ which satisfies the equation $\widetilde{P}^2 = \widetilde{P} + Id$ (the same equation as that satisfied by the golden number), and $\widetilde{g}(\widetilde{P}U, \widetilde{P}V) = \widetilde{g}(\widetilde{P}U, V) + \widetilde{g}(U, V)$ (for every tangent vector fields $U, V \in \chi(\widetilde{M})$). The pair $(\widetilde{P}, \widetilde{g})$ induces on any submanifold M of codimension r in $\widetilde{M}$, a structure similar to that above mentioned.

Finally, examples of this kind of structures induced on product of spheres of codimension r in a m-dimensional Euclidian space $(r < m)$ are constructed.

MSC2000: 53B25,53C15.

Key words: geometric structures, induced structures, submanifolds in Riemannian manifolds.

Dragos Hrimiuc (University of Alberta, Edmonton, Canada)
Homogeneous transformation of Finsler metrics

Stere Ianus (University of Bucharest, Bucharest, Romania)

Soren Illman (University of Helsinky, Helsinky, Finland)
Group actions and Hilbert's fifth problem

`Abstract:` In this lecture we consider Hilberts's fifth problem concerning Lie's theory of transformations groups. In this fifth problem Hilbert asks the following. Given a continuous action of a locally euclidean group G on a locally euclidean space M, can one choose coordinates in G and M so that the action is real analytic? We discuss two main affirmative solutions, and also present known counterexamples to the general question posed by Hilbert. The first, affirmative solution, is concerned with the special case when $M = G$, and this is the celebrated result from 1952, due to Gleason, Montgomery, and Zippin, which says that every locally euclidean group is a Lie group. The second affirmative solution is a more recent result, due to the speaker, which says that every Cartan (thus in particular, every proper) C^s differentiable action, $1 \leq s \leq \infty$, of a Lie group G, on a C^s manifold M, is equivalent to a real analytic action. In the lecture we present the main points of the proof of this latter result.

Radu-Sorin Iordanescu (Romanian Academy, Bucharest, Romania)
Jordan structures in differential geometry

`Abstract:` I shall firstly give a small introduction to Jordan structures and their applications.Next I shall present the important applications of Jordan structures to differential geometry,emphasinzing on that occasion the Romanian contributions.I shall point out that the geometry of Jordan structures is at present an important field research with a lot of elements to be further developed.Some open problems will be listed.This lecture will be partially based on my recent book entitled "Jordan Structures in Geometry and Physics".

Pyotr Ivanshin (N.G. Chebotarev RIMM of Kazan State University, Kazan, Russia)
Existence of Ehresmann connection on manifold foliated by locally free action of Lie group

`Abstract:` Recall first the following proposition [3]:
Theorem 1.*Let a manifold M with foliation F meet the following conditions:*
1) $\mathrm{codim} F = 1$.
2) *The foliation F is generated by locally free action of $H \cong \mathbb{R}^n$.*
3) *There exists a compact connected transversal on (M, F).*
Then there exist a transversal homotopic to H-complementary one and an Ehresmann connection on (M, F).
The present work is dedicated to generalisation of this theorem to the case of noncommutative Lie groups. Note that structure of proper group actions on spaces of nonpositive Alexandrov curvature as described in [2] in this case is more specific, i.e. closedness of the transversal implies that there is no purely virtually abelian factor.

Moreover the existence of the maximal tori for any Lie group [1] allows us to prove the following statement:

Theorem 2. *Let (M, F) be manifold with codimension 1 foliation generated by action of the compact Lie group G. Let (M, F) possess compact complete transversal P. Then there exists Ehresmann connection on (M, F).*

Note that the connection constructed in this statement is not necessarily G-complemetary.

Next recall the topological (Maltsev or Iwasawa) representation of the arbitrary Lie group as product $H = G \times E$ (it is not group homomorphism). Here G is maximal compact subgroup of H and E is Euclidean space. Thus the next task is to describe foliations with leaves covered by E.

Statement 1. *Assume that the foliated manifold (M, F) is such that*

1) Each leaf $L \in F$ is covered by E and is a topological group.

2) The foliation F is transversally orientable.

3) There exists a complete transversal P on (M, F).

Then there exists an Ehresmann connection on (M, F).

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Josef Janyska (Masaryk University, Brno, Czech Republic)

Natural and gauge-natural bundles and natural Lagrangian structures

`Abstract:` By using methods of natural and gauge-natural bundles we study higher order invariant (natural) Lagrangians defined on natural and gauge-natural bundles. Special attention is devoted to Lagrangians given naturally by connections, namely we present the higher order Utiyama reduction method for principal connections.

Wlodzimierz Jelonek (Cracow Technical University, Cracow, Poland)

Gray four dimensional manifolds

`Abstract:` We present compact examples of four dimensional Riemannia and semi-Riemannian manifolds whose Ricci tensor satisfies the Gray condition. Our construction is on the sphere bundles over Riemannian surfaces of arbitrary genus g. We also give partial classification of such manifolds, we classify Gray manifolds (M, g) which admit two orthogonal Hermitian structures compatible with both different orientations of M.

Igor Kanattsikov (TU, Berlin, Germany)

On the generalization of the Dirac bracket in De Donder-Weyl precanonical

Hamiltonian formalism

Spiro Karigiannis (University of Oxford, Oxford, Great Britain)
Geometric Flows on Manifolds with G_2 or Spin(7)-structure

`Abstract:` I will discuss properties of naturally defined flows of G_2 and Spin(7)-structures on manifolds. These flows are necessarily more non-linear than Ricci Flow. Study of these flows leads to new Bianchi-type identities for such structures.

Jerzy Kijowski (Centrum Fizyki Teoretycznej PAN, Warsaw, Poland)
Tomography, wave equation and symplectic geometry

`Abstract:` A simple integral transformation will be defined in the set of functions defined on a sphere $B(0, \mathbb{R})$ in the euclidean 3-space. To find the inverse transformation formula, the symplectic structure in the space of solutions of the wave equation will be used. Our problem sheds new light on the singular initial-value problem and the Trautman- Bondi theory of radiation.

Kazuyoshi Kiyohara (Okayama University, Okayama, Japan)
Jin-ichi Itoh
The cut loci and the conjugate loci on ellipsoids and some Liouville manifolds

`Abstract:` It is well known that the geodesic flow of any ellipsoid is completely integrable in the sense of hamiltonian mechanics. In this talk, we would like to discuss much finer properties of the behavior of geodesics. In particular, we shall show that the cut locus of a general point is an embedded $(n-1)$-ball ($n = \dim$ of the ellipsoid) and that the conjugate locus of a general point contains just three connected components of singularities, each of which is a cuspidal edge ($n > 2$). The latter result is a higher dimensional version of "the last geometric statement of Jacobi", which asserts that the conjugate locus of a general point on any two-dimensional ellipsoid contains just four cusps. Also, we shall show that the distribution of the conjugate points along a general geodesic possesses an interesting asymptotic property. The above results also hold for some Liouville manifolds.

Ivan Kolar (Masaryk University, Brno, Czech Republic)
Connections on principal prolongations of principal bundles

`Abstract:` We study principal connections on the r-th order principal prolongation W^rP of a principal bundle $P(M,G)$, $\dim M = m$. The bundle $W^rP \to M$, which is a principal bundle with structure group W^r_mG, plays a fundamental role both in the geometric theory of jet prolongations of associated bundles and in the gauge theories of mathematical physics. If $G = \{e\}$ is the one-element group, then $W^r(M \times \{e\}) = P^rM$ is the r-th order frame bundle of the base M. So some our results can be viewed as a generalization of the theory of connections on P^rM. In particular, there is a canonical $\mathbb{R}^m \times \mathrm{Lie}\,(W^{r-1}_mG)$-valued 1-form Θ_r on W^rP. The torsion of a connection Δ on W^rP is the covariant exterior differential $D_\Delta\Theta_r$.

Let $EP = TP/G$ be the Lie algebroid of P. Its r-jet prolongation $J^r(EP \to M)$ coincides with the Lie algebroid of W^rP. We start from the fact that the connections Δ on W^rP are in bijection with the linear splittings $\delta : TM \to J^r(EP)$. The torsion $\tau(\delta)$ of δ can be defined by means of the jet prolongation of the bracket of EP. We deduce that $D_\Delta\Theta_r$ and

$\tau(\delta)$ are naturally equivalent. Further, analogously to the case of P^rM, the torsion free connections on W^rP are in bijection with the reductions of $W^{r+1}P$ to the subgroup $G^1_m \times G \subset W^{r+1}_m G$.

According to the general theory, every principal connection Γ on P and a linear r-th order connection $\Lambda_r : TM \to J^rTM$ induce, by means of flows, a connection $\mathcal{W}^r(\Gamma, \Lambda_r)$ on W^rP. We clarify that $\mathcal{W}^r(\Gamma, \Lambda)$ can be easily constructed in the algebroid form. This enables us to deduce several original geometric results. – We also extend some our constructions and results to an arbitrary fiber product preserving bundle functor on the category of fibered manifolds with m-dimensional bases and fibered manifold morphisms with local diffeomorphisms as base map.

Mayuko Kon (Hokkaido University, Hokkaido, Japan)

Compact minimal CR submanifolds of a complex projective space with positive Ricci curvature

`Abstract:` We give a reduction theorem of the codimension of a compact n-dimensional minimal proper CR submanifold M of a complex projective space. We prove that if the Ricci curvature of M is equal or greater than n minus 1, then M is a real hypersurface of some totally geodesic complex projective space. Using this result, we classify compact minimal CR submanifolds of a complex projective space of which the Ricci tensor satisfies certain conditions.

Anatoly Kopylov (Sobolev Institute of Mathematics, Novosibirsk, Russia)

Unique determination of domains

`Abstract:` This survey lecture is devoted to relatively recent results closely connected with some 200 years old classical problems.

The starting point is a familiar Cauchy theorem *about the unique determination of convex polyhedrons (in Euclidean 3-space $\mathbb{R}^3$) by their unfoldings*. Later the problems of unique determination of convex surfaces were studied by Minkowski, Hilbert, Weyl, Blashke, Cohn-Vossen and other prominent mathematicians. Yet the greatest success was achieved by A.D. Aleksandrov and his school. We want to mention the following classical theorem by A.V. Pogorelov [1]: *if two bounded closed convex surfaces in $\mathbb{R}^3$ are isometric in their inner metrics, then these surfaces are congruent, i.e., one of them can be translated to the other by a motion*.

A new development of the subject is due to A.P. Kopylov. Kopylov offered a new approach [2], which essentially extends the scope of the above problems. He suggested to study the unique determination of domains by relative metrics of their boundaries, i.e., the metric on the boundary is defined as a continuation of the inner metric of the domain. So, the foregoing classical problems are special cases of the problem of the unique determination of domains by relative metrics of their boundaries, namely, when the complementary sets of the domains are convex sets. Moreover, a new class of very interesting problems appears in Kopylovs approach. These new problems were studied by A.D. Aleksandrov and also by V.A. Aleksandrov, M.K. Borovikova, A.V. Kuzminykh, M.V. Korobkov and others (in this connection, see, e.g., [2][12]). They discovered the following new phenomena: domains are uniquely determined not only in the classical cases (when the complementary sets of the domains are convex bounded sets), but also when domains are convex and bounded [2]; strictly convex (A.D. Aleksandrov); bounded with piece-

smooth boundary [3]; having non-empty bounded complements, where their boundaries are connected smooth $(n-1)$-manifold without edge [4]; and others.

In 2006, A.P. Kopylov considered a new unique determination problem of conformal type and proves the following assertion (see [13] and [14]): *if* $n \geq 4$, *then any bounded convex polyhedral domain* $U \subset \mathbb{R}^n$ *is uniquely determined by the relative conformal moduli of its boundary condensers in the class of all bounded convex polyhedral domains* $V \subset \mathbb{R}^n$.

We also discuss some problems related to the theory of the unique determination of the isometric and conformal types of domains in R^n.

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Julius Korbas (Comenius University, Bratislava, Slovakia)
On the cup-length and Lyusternik-Shnirel'man category

`Abstract:` The cup-length over a field R, $\mathrm{cup}_R(X)$, of a path connected space X is the supremum of all c such that there are positive dimensional cohomology classes $a_1, \ldots, a_c \in H^*(X;R)$ such that their cup product $a_1 \cup \cdots \cup a_c$ does not vanish. The cup-length is related to another important numerical invariant: we have $\mathrm{cat}(X) \geq 1+\mathrm{cup}_R(X)$, where $\mathrm{cat}(X)$ is the Lyusternik-Shnirel'man category of the space X, invented originally in variational calculus. We recall that $\mathrm{cat}(X)$ is defined to be the least positive integer k such that X can be covered by k open subsets each of which is contractible in X. If no such integer exists, then we define $\mathrm{cat}(X) = \infty$.

The aim of our talk is to present several recent results on the cup-length and Lyusternik-Shnirel'man category.

Oldrich Kowalski (Charles University, Prague, Czech Republic)

Wojciech Kozlowski (University of Lodz, Lodz, Poland.)
Collapse of unit horizontal bundles

Laszlo Kozma (Debrecen University, Debrecen, Hungary)
Sub–Finslerian geometry

`Abstract:` In the talk first the notion and basic questions of sub–Finslerian geometry will be explained. A sub-Finslerian manifold is, roughly speaking, is manifold M which is equipped with a cone (or distribution) $D \subset TM$ and sub-Finslerian function $F : D \to R^+$ with some regularity conditions. This notion, initiated by C. López and E. Martínez in 2000, generalizes the notion of sub–Riemannian geometry, called also Carnot–Carathéodory theory. We discuss how it is possible to introduce the parallelism, covariant derivations and geodesics in this circumstance. Regular and abnormal geodesics will be explained in details. Then we show the relationships and the applicability of this new notion to control theory.

Michael Krbek (Masaryk University, Brno, Czech Republic)

Boris Kruglikov (University of Tromso, Tromso, Norway)
Multi-brackets of differential operators: applications to geometric problems and exact solution methods

Demeter Krupka (Palacky University, Olomouc, Czech Republic)
Higher order Grassmann bundles

`Abstract:` The concept of a higher order Grassmann bundle is introduced within the theory of jet prolongations of smooth manifolds. Basic properties of Grassmann bundles are discussed.

Olga Krupkova (Palacky University, Olomouc, Czech Republic)
Geoff Prince (La Trobe University, Melbourne, Australia)
Lepage forms, closed two-forms and second order ordinary differential equations

`Abstract:` Lepage two-forms appear in the variational sequence as representatives of the classes of two-forms. In the theory of ordinary dif-

ferential equations on jet bundles they are used to construct exterior differential systems associated with the equations and to study solutions, and help to solve the inverse problem of the calculus of variations: as it is known, variational equations are characterized by Lepage two-forms that are closed. In this talk, a general setting for Lepage forms in the variational sequence is presented, and Lepage two-forms in the theory of second-order differential equations in general and of variational equations in particular, are investigated in detail.

Svatopluk Krysl (Charles University, Prague, Czech Republic)

Symplectic Dirac operators on complexified compactified Minkowski space

`Abstract:` We compute the spectrum of the second order operator associated to a symplectic Dirac operator on the complexified compactified Minkowski space using techniques of representation theory (universal Casimir operators and branching rules).

Jan Kubarski (Technical University of Lodz, Lodz, Poland)

Signature of transitive Lie algebroids

`Abstract:` For each transitive Lie algebroid $(A, [\![\cdot,\cdot]\!], \#_A)$ with the anchor $\#_A : A \to TM$, over n-dimensional compact oriented manifold M and n-dimensional structure Lie algebras $\boldsymbol{g}_{|x}$ [then $\operatorname{rank} A = m + n$], the following conditions are equivalent (Kubarski-Mishchenko, 2004 [K-M2])

(1) $\mathbf{H}^{m+n}(A) \neq 0$,

(2) $\mathbf{H}^{m+n}(A) = \mathbb{R}$ and $\mathbf{H}(A)$ is a Poincaré algebra,

(3) there exists a global non-singular invariant [with respect to the adjoint representation of A] cross-section ε of the vector bundle $\bigwedge^n \boldsymbol{g}$ ($\boldsymbol{g} = \ker \#_A$),

(4) $\boldsymbol{g}$ is orientable and the module class $\theta_A = 0$.

The condition (3) implies that $\boldsymbol{g}_{|x}$ are unimodular and A is the so-called TUIO-Lie algebroid (Kubarski, 1996 [K1]). The scalar Poincaré product

$$\mathcal{P}_A^i : \mathbf{H}^i(A) \times \mathbf{H}^{m+n-i}(A) \to \mathbb{R},$$

$$(\alpha, \beta) \longmapsto \int_A^{\#} \alpha \wedge \beta$$

is defined via the fibre integral $\fint_A : \Omega^{\bullet}(A) \to \Omega_{dR}^{\bullet - n}(M)$ by the formula

$$\int_A^{\#} \alpha \wedge \beta = \int_M \left(\fint_A^{\#} \alpha \wedge \beta \right).$$

The condition (3) implies that the operator $\fint_A$ commutes with the differentials d_A and d_M giving a homomorphism in cohomology $\fint_A^{\#}$: $\mathbf{H}^{\bullet}(A) \to \mathbf{H}_{dR}^{\bullet - n}(M)$. In particular we have $\fint_A^{\#} : \mathbf{H}^{m+n}(A) \to \mathbf{H}_{dR}^{m}(M) = \mathbb{R}$. The scalar product $\mathcal{P}_A^i$ is nondegenerated and if $m + n = 4k$ then

$$\mathcal{P}_A^{2k} : \mathbf{H}^{2k}(A) \times \mathbf{H}^{2k}(A) \to \mathbb{R}$$

is nondegenerated and symmetric. Therefore its signature is defined and is called the signature of A, and is denoted by $\operatorname{Sig}(A)$.

The problem is:

- to calculate the signature $\operatorname{Sig}(A)$ and give some conditions to the equality $\operatorname{Sig}(A) = 0$.

My talk concerns this problem.

(I) Firstly, I give a general mechanism of the calculation of the signature via spectral sequences (Kubarski-Mishchenko [K-M1]). Namely, under

some simple regularity assumptions on a DG-algebra C^r we have: if $E_2^{j,i}$ leave in a finite rectangular and is a Poincaré algebra then

$$\mathrm{Sig}\,(E_2) = \mathrm{Sig}\,(\mathbf{H}\,(C))\,.$$

We use this mechanism to

(a) the spectral sequence for the Čech-de Rham complex of the Lie algebroid A proving that the trivial monodromy implies $E_2^{j,i} = \mathbf{H}^j\,(M) \otimes \mathbf{H}^i\,(\mathfrak{g})$ [$\mathfrak{g} = \boldsymbol{g}_{|x}$] and so $\mathrm{Sig}\,(A) = 0$ (such a situation take place for example if the structure Lie algebra $\mathfrak{g}$ is simple algebra of the type $B_l, C_l, E_7, E_8, F_4, G_2$).

(b) the Hochschild-Serre spectral sequence. For this sequence $E_2^{j,i} = \mathbf{H}^j\left(M, \mathbf{H}^i\,(\boldsymbol{g})\right)$ with respect some flat connection in the vector bundle $\mathbf{H}^i\,(\boldsymbol{g})$ which enables us to use the Hirzebruch formula.

(II) Secondly, introducting a Riemannian structure in A we can define the $*$-Hodge operator, the codifferential and the signature operator and prove the Signature Theorem.

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Jan Kurek (Maria Curie Sklodowska University, Lublin, Poland)
Wlodzimierz M. Mikulski (Jagiellonian University, Cracow, Poland)
Like jet prolongation functors of affine bundles

`Abstract:` Let $F : \mathcal{AB}_m \to \mathcal{FM}$ be a covariant functor from the category $\mathcal{AB}_m$ of affine bundles with m-dimensional bases and affine bundle maps with local diffeomorphisms as base maps into the category $\mathcal{FM}$ of fibred manifolds and their fibred maps. Let $B_{\mathcal{AB}_m} : \mathcal{AB}_m \to \mathcal{M}f$ and $B_{\mathcal{FM}} : \mathcal{FM} \to \mathcal{M}f$ be the respective base functors.

A gauge bundle functor on $\mathcal{AB}_m$ is a functor F as above satisfying:

(i) (**Base preservation**) $B_{\mathcal{FM}} \circ F = B_{\mathcal{AB}_m}$. Hence the induced projections form a functor transformation $\pi : F \to B_{\mathcal{AB}_m}$.

(ii) (**Localization**) For every inclusion of an open affine subbundle $i_{E|U} : E|U \to E$, $F(E|U)$ is the restriction $\pi^{-1}(U)$ of $\pi : FE \to B_{\mathcal{AB}_m}(E)$ over U and $Fi_{E|U}$ is the inclusion $\pi^{-1}(U) \to FE$.

A gauge bundle functor F on $\mathcal{AB}_m$ is *fiber product preserving* if for every fiber product projections $E_1 \stackrel{pr_1}{\longleftarrow} E_1 \times_M E_2 \stackrel{pr_2}{\longrightarrow} E_2$ in the category $\mathcal{AB}_m$ $FE_1 \stackrel{Fpr_1}{\longleftarrow} F(E_1 \times_M E_2) \stackrel{Fpr_2}{\longrightarrow} FE_2$ are fiber product projections in the category $\mathcal{FM}$. In other words $F(E_1 \times_M E_2) = F(E_1) \times_M F(E_2)$ modulo the restriction of (Fpr_1, Fpr_2).

The most important example of fiber product preserving gauge bundle functor on $\mathcal{AB}_m$ is the r-jet prolongation functor $J^r : \mathcal{AB}_m \to \mathcal{FM}$, where for an $\mathcal{AB}_m$-object $p : E \to M$ we have $J^r E = \{j^r_x\sigma \mid \sigma \text{ is a local section of } E,\ x \in M\}$ and for a $\mathcal{AB}_m$-map $f : E_1 \to E_2$ covering $\underline{f} : M_1 \to M_2$ we have $J^r f : J^r E_1 \to J^r E_2$, $J^r f(j^r_x\sigma) = j^r_{\underline{f}(x)}(f \circ \sigma \circ \underline{f}^{-1})$, $j^r_x\sigma \in J^r E_1$.

The first main result in this paper is that all fiber product preserving gauge bundle functors F on $\mathcal{AB}_m$ of finite order r are in bijection with so called admissible systems, i.e. systems $(V, H, t, \mathbf{1})$, where V is a finite dimensional vector space over $\mathbb{R}$, $\mathbf{1} \in V$ is an element, $H : G^r_m \to GL(V)$ is a smooth group homomorphism from $G^r_m = invJ^r_0(\mathbb{R}^m, \mathbb{R}^m)_0$ into $GL(V)$ with $H(\xi)(\mathbf{1}) = \mathbf{1}$ for any $\xi \in G^r_m$, $t : \mathcal{D}^r_m \to gl(V)$ is a G^r_m-equivariant unity preserving associative algebra homomorphism from $\mathcal{D}^r_m = J^r_0(\mathbb{R}^m, \mathbb{R})$ into $gl(V)$.

The second main result is that any fiber product preserving gauge bundle functor on $\mathcal{AB}_m$ is of finite order.

Miroslav Kures (Brno University of Technology, Brno, Czech Republic)

Bavo Langerock (Sint-Lucas Institute for Higher Education in the Sciences & Arts, Belgium)

Routhian reduction of the Tippe Top

`Abstract:` We consider a Tippe Top modeled as an eccentric sphere, spinning on a horizontal table and subject to a sliding friction. Ignoring translational effects, we show that the system is reducible using a Routhian reduction technique. The reduced system is a two dimensional system of second order differential equations, that allows an elegant and compact way to retrieve the classification of tippe tops in six groups according to the existence and stability type of the steady states.

Valentina B. Lazareva (Tver State University, Tver, Russia)

On a Blaschke problem in web theory (some example of webs formed by pencils of spheres)

`Abstract:` In every point of 3-dimensional space E^3, 4 pencils of spheres induce a configuration consisting of 4 sphere and 6 circles. So, 4 pencils of spheres form in E^3 a unique spherical 4-web W and six 3-webs each of them is formed by 2 pencils of spheres and 1 congruence of circles.

The sphere $a(x^2 + y^2 + z^2) + bx + cy + dz + e = 0$ we consider as a point (a, b, c, d, e) in projective space P^4 (Darboux representation).

Then, the pencil of spheres is straight line in P^4.

Example 1. Let S_1, S_2, S_3 be mutually ortogonal spheres, A and B be the common points of S_1,S_2, S_3. The pencils AS_1, S_1S_2, S_2S_3, AB form spherical web W_1 whose equation can be written as $1+y^2+y^2z^2+ux^2 = 0$, or after an isotopic transformation as

$$xy + uv = 1. \tag{1}$$

This web is hexagonal but not regular (parallelizable).

Example 2. Spherical 4-web W_2 formed by pencils AS_1, S_1S_2, S_2S_3, S_3B. Its equation is $xyzu - y^2 - z^2 = 1$. The web W_2 is not hexagonal.

Example 3. Spherical 4-web W_0 with the following property: every sphere of W_0 is orthogonal to a sphere S_0. In other words, the corresponding straight lines l_i, $i = 1, 2, 3, 4$, in P^4 are situated in a 3-plane π.

The web W_0 is hexagonal, but not regular. It is regular if the lines l_i form a closed cycle.

Example 4. 3-web W_4 formed by 2 elliptic pencils of spheres S_1S_2 and S_2S_3 (see Example 1) and congruence of circles generated by hiperbolic pencil of spheres AB and parabolic pencil AS_1. The equation of the web W_4 is also equation (1) where x and y are the parameters of two first families of spheres (S_1S_2 and S_2S_3), and (u, v) (mutually!) are the parameters of the congruence of circles. After isotopic transformation $\ln x \to x$, $\ln y \to y$, $\ln(1 - uv) \to -u - v$ we transform the equation (1) to the equation $x+y+u+v = 0$. So, 3-web W_4 is regular.

Remi Leandre (Universite de Bourgogne, Dijon, France)

Deformation Quantization in Infinite Dimensional Analysis

Abstract: We present a survey on various aspects of deformation quantization in infinite dimensional analysis.

Xingxiao Li (Humboldt University of Berlin, Berlin, Germany)

Haizhong Li (Tsinghua University, Beijing, China)

Variational problems in Geometry of Submanifolds)

Abstract: In this talk, we give a survey of variational problems in geometry of submanifolds, which includes geometry of r-minimal submanifolds and geometry of *Willmore* submanifolds. We propose three open problems in our talk.

Valentin Lychagin (University of Tromso, Tromso, Norway)

Symbolic syzygies and PDE compatibility

Abstract: A relation of syzygy of symbolic modules for finding of compatibility conditions for PDE system will be discussed. Concrete forms of compatibility conditions for various classes of PDE systems will be given.

Sadahiro Maeda (Saga University, Saga, Japan)

Characterization of parallel isometric immersions of space forms into space forms in the class of isotropic immersions)

Abstract: For an isotropic submanifold M^n $(n \geqq 3)$ of a space form $\widetilde{M}^{n+p}(c)$ of constant sectional curvature c, we show that if the mean curvature vector of M^n is parallel and the sectional curvature K of M^n satisfies some inequality, then the second fundamental form of M^n in $\widetilde{M}^{n+p}$ is parallel and our manifold M^n is a space form (Theorem 1).

Yoshinori Machida (Numazu College of Technology, Numazu-shi, Shizuoka, Japan)

Differential equations associated with cone structures)

Abstract: A Goursat equation is a single 2nd order PDE of parabolic type and with Monge integrability. It is constructed from a contact manifold with a Legendre cone field. In a similar way, by considering a manifold with a cone field, we can construct a single 1st order PDE associated with the cone structure. Both kinds of equations have relations to twistor theory such as the projective duality and the Lagrange-Grassmann duality.

Mikhail Malakhaltsev (Kazan State University Kazan, Russia)
De Rham - like Cohomology for Geometric Structures

Mancho Manev (University of Plovdiv, Plovdiv, Bulgaria)
On quasi-Kahler manifolds with Norden metrics

Giovanni Manno (Dip. di Matematica E. De Giorgi, Lecce, Italy)
Vladimir S. Matveev (University of Jena, Jena, Germany)
Robert Bryant
Solution of a problem of Sophus Lie

`Abstract:` We give a complete list of normal forms for the 2-dimensional metrics that admit a transitive Lie pseudogroup of geodesic-preserving transformations and we show that these normal forms are mutually non-isometric. This solves a problem posed by Sophus Lie in 1882.

Michael Markellos (University of Patras, Rion, Greece)
The harmonicity of the characteristic vector field on contact metric 3-manifolds

`Abstract:` A contact metric manifold whose characteristic vector field is a harmonic vector field is called *H-contact metric manifold*. We introduce the notion of (κ, μ, ν)*-contact metric manifolds* in terms of a specific curvature condition, where κ, μ, ν smooth functions. Then, we prove that a contact metric 3-manifold $M(\eta, \xi, \phi, g)$ is an H-contact metric manifold if and only if it is a (κ, μ, ν)-contact metric manifold on an everywhere open and dense subset of M. Moreover, it is proved that in dimensions greater than three such manifolds are reduced to (κ, μ)-contact metric manifolds. On the contrary, in dimension three such (κ, μ, ν)-contact metric manifolds exist. After the existence of (κ, μ, ν)-contact metric manifolds in dimension 3, it is natural to study the problem of classifying all three-dimensional (κ, μ, ν)-contact metric manifolds. To this direction, we give some partial answers assuming additionally that they satisfy some interesting geometric properties.

Hiroshi Matsuzoe (Nagoya Institute of Technology, Nagoya, Japan)
Generalizations of dual connections and statistical manifolds

Roman Matsyuk (Institute for Applied Problems in Mechanics and Mathematics, Lviv, Ukraine)
The inverse variational problem in 2–dimensional concircular geometry

`Abstract:` Concircular geometry is the geometry of geodesic circles. In 2–dimensional pseudoeuclidean space the following assertion is true:
Let the autonomous third-order vector differential equation

$$\mathcal{E}_i(x^j, u^j, \dot{u}^j, \ddot{u}^j) = 0 \tag{1}$$

descend down to some variational problem with the (locally defined) Lagrange function $L = L(x^j, u^j, \dot{u}^j)$, and let it moreover satisfy the following requirements:

1. the system of equations (1) submits to the pseudoeuclidean symmetry;
2. the geodesic curves $\dot{\mathbf{u}} = \mathbf{0}$ enter in the set of solutions of (1);

3. Frenet curvature k_1 is constant along the solutions of (1).

Then

$$\mathcal{E}_i = \frac{e_{ij}\ddot{u}^j}{\|\mathbf{u}\|^3} - 3\,\frac{(\dot{\mathbf{u}}\cdot\mathbf{u})}{\|u\|^5}\,e_{ij}\dot{u}^j + m\,\frac{\|\mathbf{u}\|^2\dot{u}_i - (\dot{\mathbf{u}}\cdot\mathbf{u})u_i}{\|\mathbf{u}\|^3} \tag{2}$$

with local Lagrange function

$$L = \frac{e_{ij}u^i\dot{u}^j}{\|\mathbf{u}\|^3} - m\,\|\mathbf{u}\|\,. \tag{3}$$

We generalize this Lagrange function to the true pseudoriemannian space.

Vladimir S. Matveev (Math Institute, University of Jena, Jena, Germany)
Solution of Lie, Beltrami and Schouten problems

`Abstract:` Can two different metrics have the same geodesics? Yes! The first examples were constructed already by Lagrange, and different versions of the question were actively studied by virtually all differential geometers 100 years ago. During my talk I will explain the solution of the Lie Problem (which is the infinitesimal version of the question above; this is a joint result with R. Bryant and G. Manno), of the Beltrami Problem (which is presicely the question above, my contribution is to solve it on closed manifolds), and of the Lichnerowicz-Obata conjecture (which suggests an answer to Schouten problem).

Sergei Merkulov (Stockholm University, Stockholm, Sweden)
Poisson and Nijenhuis Geometries as Graph Complexes

Josef Mikes (Palacky University, Olomouc, Czech Republic)
On geodesic mappings and their generalizations

Velichka Milousheva (Bulgarian Academy of Sciences, Sofia, Bulgaria)
Georgi Ganchev
On the Theory of Surfaces in the Four-Dimensional Euclidean Space

`Abstract:` For a two-dimensional surface M^2 in the four-dimensional Euclidean space $\mathbb{E}^4$ we introduce an invariant linear map of Weingarten type in the tangent space of the surface, which generates two invariants k and $\varkappa$.

The condition $k = \varkappa = 0$ characterizes the surfaces consisting of flat points. We characterize the minimal surfaces by the equality $\varkappa^2 - k = 0$, and the class of the surfaces with flat normal connection by the condition $\varkappa = 0$. For the surfaces of general type we obtain a geometrically determined orthonormal frame field at each point and derive Frenet-type derivative formulas.

We apply our theory to the class of the rotational surfaces in $\mathbb{E}^4$, which prove to be surfaces with flat normal connection, and describe the rotational surfaces with constant invariants.

Chayan Kumar Mishra (Dr.R.M.L.Avadh University,Faizabad (U.P.) India)
Symmetric and Ricci$-$Symmetric Finsler spaces with Concircular Transformations

`Abstract:` Concircular transformations introduced by K.Yano[12] in Riemannian geometry; and later, extended by K.Takano[10] to affine geometry with recurrent curvature were further studied by M. Okumura[7] in different types of Riemannian Manifolds. K.Takano^{10-v} also discussed various alternative forms for the covariant derivative of the generator of and infinitesimal transformation which were; later, extended by R.B.Misra et al[5] to Finslerian manifolds of recurrent curvature; particular cases of these transformations such as Contra, Concurrent and special Concircular transformations defining projective motions were also studied by R.B. Misra[4] in a recurrent Finsler Manifolds.

Presently, we discuss the existence of a concircular infinitesimal transformation of a Symmetric and Ricci-symmetric Finsler manifolds.

Oleg Mokhov (Russian Academy of Sciences, Moscow, Russia)

Submanifolds in Pseudo-Euclidean Spaces, Associativity Equations in 2D Topological Quantum Field Theories, and Frobenius Manifolds

`Abstract:` We prove that the associativity equations of two-dimensional topological quantum field theories are very natural reductions of the fundamental nonlinear equations of the theory of submanifolds in pseudo-Euclidean spaces (namely, the Gauss equations, the Peterson–Codazzi–Mainardi equations and the Ricci equations) and determine a natural class of *potential* flat submanifolds without torsion. We show that all potential flat torsionless submanifolds in pseudo-Euclidean spaces possess natural structures of Frobenius algebras on their tangent spaces. These Frobenius structures are generated by the corresponding flat first fundamental form and the set of the second fundamental forms of the submanifolds (in fact, the structural constants are given by the set of the Weingarten operators of the submanifolds). We prove that each N-dimensional Frobenius manifold can locally be represented as a potential flat torsionless submanifold in a $2N$-dimensional pseudo-Euclidean space. By our construction this submanifold is uniquely determined up to motions. Moreover, we consider a nonlinear system, which is a natural generalization of the associativity equations, namely, the system describing the class of all flat torsionless submanifolds in pseudo-Euclidean spaces, and prove that this system is integrable by the inverse scattering method.

Giovanni Moreno (Universita di Salerno, Salerno, Italy)

Cohomological Interpretation of Transversality Conditions

Piotr Mormul (Warsaw University, Warsaw, Poland)

Cartan prolongations of distributions produce a cornucopia of singularities adjoining classical contact systems

`Abstract:` E. Cartan proposed a special way of producing more involved distributions from simpler ones. In his work [C] he was prolonging rank-2 distributions; in the most clear way his procedure was (much later) presented in the paper [BH]. Cartan's prolongations of rank-2 distributions yield new rank-2 ones that are far from being generic (most often, their growth vectors start from $[2, 3, 4, \dots]$). And precisely because of that they are very useful for distributions, called traditionally Goursat, that generate 1-flags.

In fact, any r such prolongations started on $T\mathbb{R}^2$ **produce**, along with singularityless contact distributions [called also Cartan's and originally

coming from the jet space $J^r(1, 1)$], also **all existing singularities** of Goursat flags of that length r. Namely, that resulting universal Goursat distribution D lives on a huge 'monster' $(r+2)$-manifold Mon_r and the standard contact geometry is realized by D at generic points of Mon_r, while the singularities of Goursat materialize in D at the remaining points of Mon_r.

The procedure of [C] and [BH] has been generalized in [M] to multi-dimensional, or *generalized* Cartan prolongations (gCp), that produce more involved rank-$(m+1)$ distributions from simpler, also rank-$(m+1)$, ones. The outcomes of gCp's are not generic, neither, making that operation an ideal tool for describing *special m-flags*: distributions having their flags growing in ranks always by m and having a very regular substructure – an involutive subflag similar to the one possessed automatically by 1-flags. (Multi-flags were brought into the mathematical usage by A. Kumpera in 1998; cf. p. 160 in [M]. Recently, independently in [A] and [SY], they were given a new and most compact definition.)
Special m-flags, likewise Goursat flags, appear to be but the outcomes of sequences of gCp's started from the tangent bundle to $\mathbb{R}^{m+1}$ and living on, bigger than for Goursat, monster manifolds. For any r the relevant manifold is stratified, in function of the geometry of the universal flag structure it bears, into *singularity classes*. The only generic stratum materializes contact Cartan distributions originally known from the jet spaces $J^r(1, m)$. And adjacent to it is a true cornucopia of thinner and thinner strata built of singular (for that flag structure) points of different degrees of degeneration. For length 4 there are 14 singularity classes in width 2 and 15 such classes from the width 3 onwards. To give an idea of the awaiting crowd, let us for example fix the length $r = 7$. Then the numbers of different singularity classes of special m-flags, for $m \in \{1, 2, \ldots, 6\}$, are as follows:

m	1	2	3	4	5	6
#	32	365	715	855	876	877

The value 32 is just 2^{7-2} and belongs still to Goursat world. For inst., the third value in the table, 715, is the evaluation at $r = 7$ of the formula

$$2^{r-1} + \left(2^{r-2} - 1\right)4^0 + \left(2^{r-3} - 1\right)4^1 + \cdots + \left(2^1 - 1\right)4^{r-3}$$

for the number of singularity classes of length $r \geq 4$ of special 3-flags (for $r = 4$ one gets, mentioned above, 15). This is just an example. For a general width $m \geq 2$, one should count the # of words $j_1.j_2 \ldots j_r$ over the alphabet $\{1, 2, \ldots, m, m+1\}$, $r \geq m+1$, such that $j_1 = 1$ and, for $l = 1, 2, \ldots, r-1$, if $j_{l+1} > \max(j_1, \ldots, j_l)$, then $j_{l+1} = 1+\max(j_1, \ldots, j_l)$ (the rule of the least possible *new* jumps upwards in the admissible words). That count is tractable, if unexpectedly involved.

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Jaime Munoz-Masque (CSIC, Madrid, Spain)

Peter T. Nagy (Debrecen University, Debrecen, Hungary)
On Moufang 3-webs

Nobutada Nakanishi (Gifu Keizai University, Ogaki, Japan)
Integrable Differential Forms and Its Applications

`Abstract:` It is an interesting and largely open problem to classify integrable differential p-forms with coefficients of homogeneous degree, which we simply call "homogeneous integrable differential p-forms". We especially treat here " the quadartic case", by using the result of J-P.Dufour and N.T Zung. (They classified "linear integrable differential p-forms.)

Our result has a direct application for the classification of quadratic Nambu-Poisson structures.

Bent Oersted (Aarhus University, Denmark)
Analysis and Representation Theory in Parabolic Geometries

Takashi Okayasu (Ibaraki University, Japan)
A construction of complete hypersurfaces with constant scalar curvature

`Abstract:` We only have few examples of complete hypersurfaces with constant positive scalar curvature in the euclidean spaces.

1) spheres,
2) products of spheres with the euclidean subspaces,
3) rotational hypersurfaces.

The purpose of this talk is to construct a new family of complete hypersurfaces with constant positive scalar curvature in the euclidean spaces by the method of equivariant differential geometry.

Peter Olver (University of Minnesota, USA)
Moving Frames, Differential Invariants, and Applications

Nadejda Opokina (Kazan State University, Kazan, Russia)

Radomir Palacek (University of Ostrava, Ostrava, Czech Republic)

Marcella Palese (University of Torino, Torino, Italy)
Covariant field theories

`Abstract:` We prove that, for the correct description of covariant field theories, the geometric framework of gauge-natural bundles must be restricted to a specific subcategory characterized by a variational PDE. Applications to spinor fields coupled with gravity are worked out.

Martin Panak (MU AVCR, Brno, Czech Republic)

Bochner-Kaehler and special symplectic geometries

Abstract: The Bochner-Kaehler geometries are the special case of special symplectic geometries. We obtain a (local) classification of these based on the orbit types of the adjoint action in $\mathrm{su}(n, 1)$. The relation between Sasaki and Bochner-Kaehler metrics in cone and transveral metrics constructions is discussed. The duality between Bochner-Kaehler and Ricci-type geometries is outlined. There is also a connection between some of the special symplectic geometries and Weyl structures.

Jeong Hyeong Park (Sungkyunkwan University, Suwon, Korea)
Peter Gilkey (University of Oregon, Eugene, OR, USA)
C. Dunn
Spectral geometry of the Riemannian submersion of a compact Lie Group

Abstract: Let G be a compact connected Lie group which is equipped with a bi-invariant Riemannian metric. Let $m(x, y) = xy$ be the multiplication operator. We show the associated fibration $m : G \times G \to G$ is a Riemannian submersion with totally geodesic fibers and we study the spectral geometry of this submersion. We show the pull back of eigenforms on the base have finite Fourier series on the total space and we give examples where arbitrarily many Fourier coefficients can be non-zero. We give necessary and sufficient conditions that the pull back of a form on the base is harmonic on the total space.

Ales Patak (Masaryk University, Brno, Czech Republic)
The Hilbert-Yang-Mills functional: Examples

Abstract: The geometric structure of gauge natural theory is investigated.

Daniel Peralta-Salas (Universidad Carlos III de Madrid, Madrid, Spain)
Alberto Enciso (Universidad Complutense de Madrid, Madrid, Spain)
On the topology of the level sets of harmonic functions in Euclidean spaces

Abstract: We say that a function $f : \mathbb{R}^n \to \mathbb{R}$ is harmonic if $\Delta f = 0$, Δ standing for the standard Laplace operator. The aim of this talk is to sketch the proof of the following theorem that we have recently obtained in collaboration with Alberto Enciso (UCM):

Theorem 1 *Any open codimension 1 algebraic submanifold $L \subset R^n$ is diffeotopic to a non-singular component of the zero-level of some harmonic function in R^n. Under suitable conditions on L the diffeotopy can be taken arbitrarily close to the identity.*

We will also discuss some generalizations of this result, e.g. we will provide examples of harmonic functions with level sets not homeomorphic to an algebraic set and we will study (singular) foliations defined by harmonic functions, in particular foliations by planes. This work (joint with Alberto Enciso) is still in progress and provides partial answers to a very hard problem posed by Lee Rubel in 1988 (Lect. Notes Math. 1344): is any continuous function in R^n homeomorphic to a harmonic function?

Monika Pietrzyk (Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany)
Igor Kanattsikov (TU, Berlin, Germany)
Multisymplectic analysis of the Short Pulse Equation

Abstract: The so called Short Pulse Equation (SPE) [1] has received attention in nonlinear optics research recently as an alternative to the description of ultrashort optical pulses in nonlinear media beyond the standard slowly varying envelope approximation and the nonlinear Schroedinger equation. SPE is found to be integrable. In [2] we have found its two component integrable generalization. Here we present a multisymplectic (De Donder - Weyl) description of SPE and its integrable two component generalization. We use it in order to construct an effective multisymplectic numerical scheme and present some results of numerical simulations which compare the effectiveness of the standard (pseudo-spectral) and the multisymplectic numerical integrations of SPE.
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Liviu Octavian Popescu (University of Craiova, Craiova, Romania)
Dragos Hrimiuc (University of Alberta, Edmonton, Canada)
Geometrical structures on Lie algebroids and applications

Abstract: In this paper we investigate the geometry of the prolongation of Lie algebroid over the vector bundle projection of a dual bundle. The Ehresmann connection, torsion and curvature, tangent and complex structure are studied. The connection associated to a regular section is obtained and the homogeneous case is investigated. Finally, a problem of optimal control (distributional system) is solved using the framework of a Lie algebroid.

Andrey Popov (Moscow State University, Moscow, Russia)
Discrete nets on the Lobachevsky plane and their applications

Abstract: Methods of non-Euclidean hyperbolic geometry are applied to the problem of construction of the numerical integration algorithms for some types of nonlinear equations in modern mathematical physics. The methodology of the approach is based on the theory of Λ^2-representations of differential equations [1,2] which associates them with metrics of constant negative curvature. Based on an analysis of a discrete rhombic Tchebychev net on the Lobachevsky plane, a well-constructed geometric algorithm for solving of the Darboux problem for the sine-Gordon equation is presented. Some problems arising in the context of the present investigation are discussed [3,4].

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Serge Preston (Portland State University, Portland, OR, USA)
Multisymplectic Theory of Balance Systems and the Entropy Principle

`Abstract:` In this talk we are presenting the theory of balance equations of the Continuum Thermodynamics (balance systems) in a geometrical form using Poincare-Cartan formalism of the Multisymplectic Field Theory. A constitutive relation $\mathcal{C}$ of a balance system $\mathcal{B}_{\mathcal{C}}$ is realized as a mapping between a (partial) 1-jet bundle of the configurational bundle $\pi : Y \to X$ and the extended dual bundle similar to the Legendre mapping of the Lagrangian Field Theory. Invariant (variational) form of the balance system $\mathcal{B}_{\mathcal{C}}$ is presented in three different forms and the space of admissible variations is defined and studied. Action of automorphisms of the bundle π on the constitutive mappings $\mathcal{C}$ is studied and it is shown that the symmetry group $Sym(\mathcal{C})$ of the constitutive relation $\mathcal{C}$ acts on the space of solutions of balance system $\mathcal{B}_{\mathcal{C}}$. Suitable version of Noether Theorem for an action of a symmetry group is presented with the usage of conventional multimomentum mapping. The "entropy principle" of Irreducible Thermodynamics - requirement that the entropy balance should be the consequence of the balance system is studied for a general balance systems. The structure of corresponding (secondary) balance laws of a balance system $\mathcal{B}_{\mathcal{C}}$ is studied for the cases of different (partial) 1-jet bundles as domains of the constitutive relations. Corresponding results may be considered as a generalization of the transition to the dual, Lagrange-Liu picture of the Rational Extended Thermodynamics (RET). Finally, the geometrical (bundle) picture of the RET in terms of Lagrange-Liu fields is developed and the entropy principle is shown to be equivalent to the holonomicy of the current component of the constitutive section.

Geoff Prince (La Trobe University, Australia)
When is a connection Finsler?

Fabrizio Pugliese (Universita di Salerno, Salerno, Italy)
Giovanni Manno (Dip. di Matematica E. De Giorgi, Lecce, Italy)
R. Alonso-Blanco
Normal forms for degenerate Monge-Ampere equations

`Abstract:` By applying simple properties of contact geometry of first order jet bundle, normal forms of degenerate Monge-Ampere equations are found.

Christof Puhle (Humboldt University of Berlin, Berlin, Germany)
The Killing equation with higher order potentials

Morteza M. Rezaii (Amirkabir University of Technology, Tehran, Iran)
Induced Deformed Finsler Metrics And Deformed Non-linear Connections

`Abstract:` In this paper we study the induced deformed non-linear connections on Finsler manifolds. We show that there is a one to one corespondance between the set of non-linear connections on a Finsler manifold and its Finsler submanifolds. The induced deformed Finsler metrics related to induced deformed non-linear connection are also studied.

Vladimir Rovenski (University of Haifa, Haifa, Israel)

Integral fomulae on foliated Riemannian manifolds

`Abstract:` The integral formulae for foliations on Riemannian manifolds (compact or of a finite volume) have applications to the problems: a) minimizing functions like volume and energy defined for plane fields on Riemannian manifolds; b) prescribing higher mean curvatures σ_k (defined in terms of the 2-nd fundamental form) of a foliation $\mathcal{F}$ or orthogonal distribution $\mathcal{F}^\perp$.

We present new integral formulae for foliations 1) of codimension-1: they relate integrals of σ_k's with those involving some algebraic invariants obtained from A, the Weingarten operator of $\mathcal{F}$, R_N, the curvature operator in the normal direction to $\mathcal{F}$, and their products $(R_N)^m A$ **[4]**; 2) with totally geodesic leaves of arbitrary codimension involving invariants of the co-nullity operator of a foliation, the mixed curvature tensor (in the direction of the foliation and its orthogonal complement) and their products **[5]**.

Our formulae extend the results of Brito, Langevin and Rosenberg **[1]** on codimension 1 foliations of space forms, and Brito and Naveira **[2]** concerning totally geodesic foliations of arbitrary codimension on space forms.

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Alexey Rybnikov (Moscow State M.V. Lomonosov University, Moscow, Russia)

Theory of connections and Backlund maps for second-order partial differential equations

`Abstract:` In the present work the theory of Backlund transformations is treated as a special chapter of the theory of connections. Following F. Pirani and D. Robinson we consider the notion of Backlund transformation as a more general notion of *Backlund map*. We present here new interpretation of Backlund map. In this lecture Backlund maps are associated with the connections defining the representations of zero curvature for a given differential equation.

Remind first of all that Backlund transformations arose for the first time in 1879 as transformations of the surfaces with constant negative curvature in 3-dimensional Euclidean space $\mathbb{R}^3$ (wellknown BianchiLie transformation). In 1880 A. Backlund noticed that one can consider BianchiLie transformation as a particular case of the more general transfor-

mations defined by the system of four relations of the form

$$F_\alpha(x^1, x^2, z, z_1, z_2, x^{1'}, x^{2'}, y, y_{1'}, y_{2'}) = 0 \quad (\alpha = 1, 2, 3, 4),$$

where $z_i = \dfrac{\partial z}{\partial x^i}$ $(i = 1, 2)$, $y_{i'} = \dfrac{\partial y}{\partial x^{i'}}$ $(i' = 1', 2')$. Remark that A. Backlund treated these transformations as the mappings between the pairs of surfaces in $\mathbb{R}^3$. It was G. Darboux who noticed that Backlund transformation may be regarded as a mapping between integral manifolds of a pair of partial differential equations (PDE) or of a single differential equation. In 1977 F. Pirani and D. Robinson [1] presented an another interpretation of Backlund transformations. They treated the Backlund transformations as a particular case of more general notion of *Backlund maps*. In this lecture our interpretation of Backlund maps is distinguished from one of F. Pirani and D. Robinson.

We investigate here the Backlund maps for secondorder PDE

$$\Phi(x^1, x^2, z, z_1, z_2, z_{11}, z_{12}, z_{22}) = 0, \qquad (1)$$

where $z_{kl} = \dfrac{\partial^2 z}{\partial x^k \partial x^l}$. While considering the equation (1), we assumed that x^1, x^2, z are adapted local coordinates on $(2+1)$-dimensional general type bundle H over 2-dimensional base (the variables x^1, x^2 are local coordinates on the base). Along with the manifold H we consider 1-jet manifold J^1H and 2-jet manifold J^2H. Denote the local coordinates of J^1J by x^i, z, p_j and the local coordinates of J^2H by x^i, z, p_j, p_{kl} $(p_{kl} = p_{lk})$. For any section $\sigma \subset H$ defined by the equation $z = z(x^1, x^2)$ one can consider the lifted sections $\sigma^1 \subset J^1H$ and $\sigma^2 \subset J^2H$ defined by the equations $z = z(x^1, x^2)$, $p_j = z_j$ and $z = z(x^1, x^2)$, $p_j = z_j$, $p_{kl} = z_{kl}$ accordingly. A somewhat more general form of the equation (1) is

$$\Phi(x^1, x^2, z, p_1, p_2, p_{11}, p_{12}, p_{22}) = 0. \qquad (1\text{bis})$$

On the lifted section $\sigma^2 \subset J^2H$ of any section $\sigma \subset H$ this equation has the form (1). We say that a section $\sigma \subset H$ is a solution of the PDE (1) if the equation (1 bis) is identically satisfied on the lifted section $\sigma^2 \subset J^2H$.

One can consider the principal bundles over the base J^1H and over the base J^2H as well as associated bundles. Following [2] we denote $P(B, G)$ the principal bundle over the base B with the structural Lie group G. An associated bundle with the model fiber $\mathfrak{J}$ ($\mathfrak{J}$ is the representation space of the group G) we denote by $\mathfrak{J}(P(B, G))$. The bundle $^1RH = P(J^1H, SL(2))$ (it is a factor manifold of the 1-frame manifold) and the bundle $^2RH = P(J^2H, SL(2))$ (it is a factor manifold of the 2-frame manifold) are the most important for us. The bundles $P(J^kH, G)$ $(k = 1, 2)$, where G is a subgroup of $SL(2)$, are interested for us too.

One can consider the *special connections* [3] in the principal bundles $P(J^kH, G)$ $(k = 1, 2)$ $(G \subseteq SL(2))$. They generate corresponding connections in associated bundles and, in particular, in the associated bundles with one-dimensional fiber $\mathfrak{J}$. We say that a connection in $\mathfrak{J}(P(J^kH, G))$ $(\dim\mathfrak{J} = 1)$, where $(G \subseteq SL(2))$, is *Backlund connection of class* k corresponding to a given PDE (1) if it is generated by special connection defining the representation of zero curvature for the equation (1). Moreover note that in this case Pfaff equation

$$_\sigma\theta = 0 \qquad (2)$$

is completely integrable if and only if the section $\sigma \subset H$ is a solution of the equation (1). Here $_\sigma\theta$ is the connection form for the Backlund connection of the equation (1) considered on a section $\sigma \subset H$. The Pfaff equation (2) defines a mapping that takes each solution $\sigma \subset H$ of the equation (1) to the section $_\sigma\sum \subset \mathfrak{J}(P(J^kH, G))$ $(\dim\mathfrak{J} = 1)$ which is a solution of the Pfaff equation (2). We say that this mapping is *the Backlund map of class*

k corresponding to the PDE (1). The Pfaff equation (2) we call *the Pfaff equation defining the Backlund map*.

The assignment of the Backlund map of class k of general type is equivalent to the assignment of the Backlund connection in $\mathfrak{J}({}^kRH)$ $(\dim\mathfrak{J} = 1)$. Moreover here $G = SL(2)$. In this case the Pfaff equation (2) (under special choice of the structural forms) is equivalent to the system of PDE

$$y_i = -{}_\sigma\gamma^2_{1i} + y \cdot_\sigma \gamma_i + y^2 \cdot_\sigma \gamma^1_{2i} \quad (i = 1, 2), \tag{3}$$

where ${}_\sigma\gamma_i$, ${}_\sigma\gamma^2_{1i}$, ${}_\sigma\gamma^1_{2i}$ are the coefficients of the special connection in ${}^kRH$ (considered on a section $\sigma \subset H$) defining the representation of zero curvature for a given PDE (1). The coefficients γ_i, γ^2_{1i}, γ^1_{2i} depend on x^i, z, p_j if $k = 1$ and on x^i, z, p_j, p_{kl} if $k = 2$. The system (3) we call *the Backlund system*.

One can prove that the coefficients of special connection in ${}^2RH$ defining the representation of zero curvature for the secondorder PDE have the following form

$$\gamma_i = \varphi{\cdot}p_{1i}+\psi{\cdot}p_{2i}+\chi_i,\ \gamma^2_{1i} = \varphi^2_1{\cdot}p_{1i}+\psi^2_1{\cdot}p_{2i}+\chi^2_{1i},\ \gamma^1_{2i} = \varphi^1_2{\cdot}p_{1i}+\psi^1_2{\cdot}p_{2i}+\chi^1_{2i},$$

where $\varphi, \varphi^2_1, \varphi^1_2, \psi, \psi^2_1, \psi^1_2, \chi_i, \chi^2_{1i}, \chi^1_{2i}$ are the functions of the variables x^1, z, p_j. Thus the Backlund system defining the Backlund map of class 2 is essentially more specific than one defining the Backlund map of class 1.

Along with the general Backlund connections one can consider the Backlund connections defined in the associated bundles $\mathfrak{J}(P(J^kH, G))$ $(\dim\mathfrak{J} = 1)$, where G is a subgroup of $SL(2)$. In this case the Backlund maps of more special types arise. Such Backlund maps are, in particular, the Backlund maps for evolution equations [3,4]. One can consider also the Backlund connections defined in $\mathfrak{J}(P(J^kH, G_1))$ $(\dim\mathfrak{J} = 1)$, where G_1 is one-dimensional subgroup of $SL(2)$. We call these connections *the ColeHopf connections*. The corresponding Backlund maps we call *ColeHopf maps* (or *CHmaps*). The very first example of such maps is well-known transformation that takes the solutions of the Burgers equation $z_1 + z \cdot z_2 - z_{22} = 0$ to the solutions of the heat equation $y_1 = y_{22}$. This transformation has appeared at first in the works of J.D. Cole and E. Hopf (see [5] or original papers of J.D. Cole [6] and E. Hopf [7]). Remark that the assignment of the ColeHopf map is equivalent to the assignment of the potential of a given PDE.

In the present work we prove that if the secondorder PDE admits the Backlund maps of class 1 then it is quasilinear equation. If the secondorder equation admits the Backlund map of class 2 then it is either quasilinear equation or MongeAmpere type equation

$$z_{11} \cdot z_{22} - (z_{12})^2 + Pz_{11} + Qz_{12} + Rz12 + S = 0,$$

where P, Q, R, S depend on x^i, z, z_j.

It is obtained a number of conditions for existence of general Backlund maps for secondorder PDE. It is found also some conditions of existence of CHmaps (and consequently potentials). In particular, it is derived the necessary and sufficient conditions for existence of CHmaps of class 1 for the equations of the form $z_{11} + z_{22} + f(x^1, X62, z, z_1, z_2) = 0$ and for the equations of the form $z_{22} + f(x^1, x^2, z, z_1, z_2)$. Besides it is proved the existence of CHmaps of class 2 for MongeAmpere type equations of the form $z_{11} \cdot z_{22} - (z_{12})^2 + g(x) \cdot f(z_1, z_2) = 0$.

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Willy Sarlet (Ghent university, Ghent, Belgium)

Partially decoupling non-conservative systems of cofactor type

`Abstract:` Cofactor-type systems were introduced by Lundmark and Wojciechowski as a specific kind of non-conservative Newtonian systems which possess a first integral quadratic in the velocities. An interesting special case concerns systems which have a double cofactor-type representation. The same authors later studied so-called driven cofactor systems. These have the property that a number of the equations (the driving system) decouple from the rest and the remaining equations (the driven system), which become time-dependent along solutions of the driving system, are assumed in that sense to have forces derivable from a time-dependent potential. This reduced system then turns out to have a separable time-dependent Hamilton-Jacobi equation.

We discuss the geometry of partially decoupling second-order equations in general and then explain the geometric structures underlying driven cofactor systems. In doing so, we generalize the original set-up of Lundmark and Wojciechowski to allow for kinetic energy terms determined by an arbitrary Riemannian metric instead of the Euclidean one.

David Saunders (Palacky University, Olomouc, Czech Republic)

The Cartan form, twenty years on

`Abstract:` The period around twenty years ago saw significant activity in the specification of "Cartan forms" for variational field theory. As well as the positive results, there were some negative ones and also some open questions. We review these results and compare them with recent developments in the homogeneous version of the theory. An important tool in the latter theory is a bicomplex comprising spaces of vector-valued forms, and an associated global homotopy operator.

Kirill Semenov (Moscow State M.V.Lomonosov University, Moscow, Russia)

On the conditions of the existense of Backlund maps and transformations for the third order evolution-type PDEs with one dimensional variable

`Abstract:` The work is contributed to the geometrical theory of Back-

lund transformations (BT) for the third order PDE's of the evolution type with one dimensional variable:

$$z_t - f(t, x, z, z_x, z_{xx}, z_{xxx}) = 0 \qquad (1)$$

See [1, 2] on the geometrical theory of BT.

Followinf F.Pirani and D.Robinson [3], we treat the concept of the Backlund Transformations (BT) as a particular case of more general concept – Backlund Map (BM), which we understand as a connection, defining zero-curvature representation for the given evolution equation.

It is proved, that equation (1) possess BM if and only if it has a form:

$$z_t + K(z,\, z_x,\, z_{xx}) \cdot z_{xxx} + L(z,\, z_x,\, z_{xx}) = 0 \qquad (2)$$

The particular case of the equation (2) are the equations of the form: $z_t + z_{xxx} + M(z, z_x) = 0$. It is proved that these equations (2) posses BM if and only if they have a form:

$$z_t + z_{xxx} + \beta(z)(z_x)^3 + \alpha(z) \cdot z_x = 0$$

Among these equations are the equations of the form:

$$z_t + z_{xxx} + \alpha(z) \cdot z_x = 0 \qquad (3)$$

One of them is a well-known KdF-equation $z_t - 6z \cdot z_x - z_{xxx} = 0$. It is proved, that equations (3) posses BT if and only if they have a form:

$$z_t + z_{xxx} + \frac{\varphi(z)}{az + b} \cdot z_x = 0 \qquad (4)$$

The system of PDEs, which define BT (so called Backlund system) is given.

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Esra Sengelen (Istanbul Bilgi University, Istanbul, Turkey)
Fusun Ozen, **Sezgin Altay**

Torse-forming vector field in a pseudo-Ricci symmetric space

Abstract: A non-flat Riemannian space is called pseudo-Ricci symmetric and denoted by $(PRS)_n$ if the Ricci tensor is non-zero and satisfies the condition $R_{ij,k} = 2\lambda_k R_{ij} + \lambda_i R_{kj} + \lambda_j R_{ik}$ where λ_i is covariant vector(non- zero simultaneously). Pseudo-Ricci symmetric space introduced by Chaki,[1], before.

In this paper, we shall consider a torse-forming vector field v^i in pseudo-Ricci symmetric space. We may assume that

$$v^h_{,i} = \phi_i v^h + \rho \delta^h_i \qquad (1.1)$$

where ρ and ϕ are any scalar function and covariant vector field, respectively, [2], [3]. Let us take an innitesimal vector field $\bar{x} = x^i + v^i(x)\delta t$ in $(PRS)_n$ where $v^i(x)$ is a covariant vector field.

We prove that if $(PRS)_n$ with positive definite metric and has scalar curvature non-constant admits an infinitesimal pseudo-homothetic motion, then this is either a homothetic motion or a motion. If the equation (1.1) satisfy the relation $\nabla_i v^h = \rho\delta_i^h$ then this vector field is called concurrent.

Some theorems that was prove in this paper are in the following: We consider a pseudo-Ricci symmetric space (V^n, g), has positive definite metric and non-constant scalar curvature, admits a pseudo-homothetic motion.

THEOREM 1. Let (V^n, g) admit a recurrent vector field. In order that the motion be isometry, it is necessary and sucient that the vector v and λ be orthogonal to each other.

THEOREM 2. Let (V^n, g) admit a concurrent vector field. If the vector v and λ be orthogonal to each other then V_n whose streamlines are geodesics can not exist.

THEOREM 3. Let (V^n, g) admit a recurrent vector field. Then it will be isometry and the vectors v and λ are orthogonal.

THEOREM 4. Let (V^n, g) admit a concircular vector field. In order that the vector ρ_j and v_j be collinear, it is necessary and sufficient that ϕ_j and v_j be collinear. If $\varphi = \rho$ then this vector field is either recurrent or concurrent.

THEOREM 5. Let (V^n, g) admit a torse-forming vector field. In order that the vector v_i and ϕ_i be orthogonal, it is necessary and sufficient that $\varphi = \rho$.

THEOREM 6. Let (V^n, g) admit a semitorse-forming vector field. In order that the vector v_k and ϕ_k be orthogonal, it is necessary and sufficient that v_k and ρ_k be orhogonal.

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Artur Sergyeyev (Silesian University in Opava, Opava, Czech Republic)

Multiparameter Generalization of the Stackel Transform, Deformations of Separation Curves and Reciprocal Transformations

Absos Ali Shaikh (The University of Burdwan, Burdwan, India)

On Pseudo Quasi-Einstein manifold

Alexander M. Shelekhov (Tver State University, Tver, Russia)

Triangulatians of plane and sphere by 3 pencils of circles

`Abstract`: We say that a plane π is triangulated by 3 smooth foliations λ_i, $i = 1, 2, 3$, if the 3-web W formed by λ_i is hexagonal, i.e. All hexagonal configurations are closed on W (see. Fig.1). By W. Blaschke [1], 3-web W is hexagonal if it is regular, that is equivalent to a 3-web formed by 3 families of parallel straight lines.

About 1953, W. Blaschke stated the problem to find all hexagonal (reg-

ular) 3-webs formed by 3 pencils of circles on a plane or a sphere (we call sach webs circle-webs). We give a complete and transparent solution to the problem. The solution is based on the following proposition that we call the theorem on the boundaries: each smooth boundary of a regular curvilinear 3-web can be only a leaf of this web. (The boundary consists of the points in which the leafes of one of foliations of 3-web W tangent to the leafes of another foliation of W). Generally, the boundary of an arbitrary circle web is a 4-degree algebraic curve. By the theorem on the boundaries this algebraic curve is decomposed in two circles for a regular circle web W.

Using Darboux transformation we find all the cases when the above mentioned 4-degree curve is a decomposition of 2 circles belonging to 3-web W. The main result is the following: only 8 classes of hexagonal circle webs indicated in [2] exist there.

Detailed proofes see in [3].

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Vadim V. Shurygin (Kazan State University, Kazan, Russia)
Lifts of Poisson structures to Weil bundles

`Abstract:` In the present paper, we construct complete and vertical lifts of tensor fields from the smooth manifold M to its Weil bundle $T^{\mathbf{A}}M$ for the case of a Frobenius Weil algebra $\mathbf{A}$.

We show that the complete lift of exterior forms induces the homomorphism of the de Rham cohomology spaces $H^*_{dR}(M) \to H^*_{dR}(T^{\mathbf{A}}M)$.

We prove that this homomorphism is either a zero map or an isomorphism depending on the Frobenius form of $\mathbf{A}$.

For a Poisson manifold (M, w) we show that the complete lift w^C and the vertical lift w^V of a Poisson tensor w are Poisson tensors on $T^{\mathbf{A}}M$ and establish some properties of Poisson manifolds obtained. Finally, we compute the modular classes of lifts of Poisson structures.

Marie Skodova (Palacky University, Olomouc, Czech Republic)
Concircular vector fields on compact manifolds with affine connections

`Abstract:` Certain properties of torse-forming, concircular and convergent vector fields on manifolds with affine connection are studied. Connections on manifolds for such vector fields exist are found. Moreover, exampels of mentioned manifolds in case if they are compact and metrizable are presented.

Jan Slovak (Masaryk University, Brno, Czech Republic)
A nearly invariant calculus for differential invariants of parabolic geometries

`Abstract:` The classical calculus for conformal Riemannian invariants is reformulated and essentially generalized to all parabolic geometries. Instead of following Schouten's and Wnsch's computational tricks, our approach is based on the canonical Cartan connections and the Weyl connections underlying all such geometries.

Dana Smetanova (Palacky University, Olomouc, Czech Republic)
Hamiltonian systems in dimension 4

Abstract: The aim of the talk is announce some recent results concerning Hamiltonian theory. The case of first order Hamiltonian systems related to an affine second order Euler–Lagrange form is studied. In dimension 4 the structure of Hamiltonian systems (i.e., Lepagean equivalent of an Euler–Lagrange form) is found.

Dalibor Smid (Charles University, Prague, Czech Republic)
Howe duality for Rarita Schwinger representation

Abstract: It is well known that the algebra of polynomials can be decomposed as a tensor product of harmonic polynomials and the set of invariants, which is generated by symbols of powers of Laplace operator (Fisher decompostition). An analogous decomposition is available for spinor-valued polynomials, where Dirac operator replaces Laplace. We study the case of polynomials valued in Rarita-Schwinger representation, where the structure of invariants is richer and may lead to a new example of Howe duality. As usually, the action of the Howe dual pair can clarify quite a few issues connected with a study of properties of solutions of R-S equations.

Petr Somberg (Charles University, Prague, Czech Republic)
Invariants of geodesics and conformal geometry

Abstract: It has been shown by U. Semmelmann that Killing forms on a Riemannian manifold yield tensor fields preserved (covariantly constant) along geodesics. We show that this is just an example of quite general phenomenon underlying the relationship between Riemannian and conformal geometry, i.e. we define generalized Killing tensor-spinors as suitable subspaces of conformal Killing tensor-spinors and prove that their contraction with geodesic vector field is covariantly constant along geodesics.

Vladimir Soucek (Charles University, Prague, Czech Republic)
A. Cap
Curved Casimir operator for parabolic geometries

Abstract: In the homogeneous model G/P of a parabolic geometry, invariant operators act on spaces of sections of homogeneous bundles over G/P, which themselves are G-modules - so called (degenerate) principle series representations. We are going to show that the Casimir operator C (defined on these spaces of sections) extends to an invariant operator on a curved version of the parabolic geometry and we will show how to express it on some types of natural bundles using the tractor calculus for these geometries. We shall also discuss the relation to BGG complexes and applications to constructions of further invariant operators.

Karl Strambach (University of Erlangen-Nurnberg, Erlangen, Germany)

Zoltan I. Szabo (City University of New York, New York, USA)
Spectral analysis on Zeeman manifolds

Abstract: By a recent observation, the Laplace operator on the Riemannian manifolds this author used for isospectrality constructions is nothing but the Landau-Zeeman operator of finite many electrons orbiting in constant magnetic fields. In physics, this operator was used to explain Zeeman effect. The Riemann manifolds having this coincidence are called Zeeman manifolds. The most simple examples can be constructed on 2-step nilpotent Lie groups. This lecture proceeds with this case.

There is a natural representation, called Fock-Bargmann representation, of the complex Heisenberg Lie algebra on the Hilbert space of complex valued functions defined on Zeeman manifolds (the Hilbert norm is defined by a natural Gauss density). The physicists introduce the LZ-operator by means of this FB-representation and the Maxwell equations. The FB is a reducible representation. One of its irreducible subspaces is the Fock space spanned by the holomorphic polynomials. The projection onto the Fock space is an integral operator whose kernel was explicitly computed by Bargmann.

Although the FB-representation naturally acts on the total space of complex valued functions, it has been considered only on the Fock space and no thorough investigation of the extended representation is known in the literature. In this talk all the irreducible subspaces, called zones, with the corresponding projections will be explicitly determined. A thorough spectral analysis of these zones is the other objective of this talk. For instance, each zone is invariant under the action of the LZ-operator as well as flows such as the heat- resp. Schroedinger-flows. The corresponding zonal spectrum and zonal flows will also be explicitly determined. It is well known that the global objects define infinite physical quantities for the considered particles. In physics, these infinities are handled by the perturbative renormalization theory, which produces the desired finite quantities by differences of infinities. The most remarkable feature of this zonal theory is that all these quantities are finite ones on the zonal setting. This zonal spectral theory is a new non-perturbative tool by which the infinities appearing in QED can be handled.

Janos Szenthe (Eotvos Lorand University, Budapest, Hungary)
Stationary geodesics of left-invariant Lagrangians

Abstract: In 1765 L. Euler published his theory of rigid body motion [E], and in 1965 V. I. Arnold to celebrate the 200 years' anniversary of this feat generalized Euler's theory in terms of left-invariant Riemannian metrics on Lie groups and on groups of diffeomorphisms [A]. In order to study stability problems Arnold introduced the concept of *stationary geodesic*, by which such a geodesic is meant which is also a 1-parameter subgroup, also called *homogeneous geodesic* nowdays. Thus the set of the stationary geodesics of a left-invariant Riemannian metric on a compact semi-simple Lie group was studied [Sz1], [Sz2]. Later on left-invariant Lagrangians on Lie groups were considered and accordingly the concept of stationary geodesics was generalized to this case [Sz3]. In the lecture results will be presented concerning the existence and classification of stationary geodesics of left-invariant Lagrangians.

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Jozsef Szilasi (Debrecen University, Debrecen, Hungary)
Some aspects of differential theories

`Abstract:` The talk is (in some sense) an outgrowth of the survey paper written together with *R. L. Lovas* under the same title and to be published in the forthcoming *Handbook of Global Analysis* edited by *D. Krupka* and *D. Saunders*. In the paper we focused on calculus on Frchet manifolds, based on Michal–Bastiani's concept of differentiability and emphasized that such tools may be useful also for differential geometers working in finite dimensions. In the talk I would like to discuss further aspects of differential theories, touched upon in the paper.

Since *Newton* it has been known that there is (or there must be) a strong and deep connection between physics ('natural philosophy'), geometry and calculus: a glimpse of their historical development makes clear the mutually enriching interaction between them. (Quite paradoxically, an adequate geometrical formulation of classical mechanics was achieved only in the 1950s.) Integrating physical content, geometric framework and appropriate calculus in a unified theory remains a permanent challenge and endeavour; it is enough to refer to the recent efforts of *N. Bourbaki*'s civilian relative, *Jet Nestruev*. Less ambitiously, in the talk I would like to sketch a unified theory for a more restricted area, which still covers a significant part of geometry, analysis and physics.

The story starts in the early 1920s, when an intensive study of the geometry of manifolds endowed with a 'system of paths' given by the Newtonian SODE

$$x^{i''} + 2G^i(x, x') = 0 \quad (i \in \{1, \ldots, n\})$$

began. (It is assumed here that the functions G^i are positively homogeneous of degree 2 and smooth on the slit tangent bundle of the underlying manifold. For simplicity, I also suppose that the base manifold can be covered by a single coordinate system.) Based on this SODE, and using the tools of classical tensor calculus, a rich geometry, devided into the classes 'general', 'affine' and 'projective' geometry of paths was elaborated in the short period 1920-1930, due to the efforts of such eminent mathematicians as *L. Berwald*, *E. Cartan*, *J. Douglas*, *T. Y. Thomas*, *O. Veblen*, *H. Weyl* and others. The concept of a spray (Ambrose–Palais–Singer, 1960) and its near mutations, combined with an index-free construction of linear and non-linear connections made possible a reincarnation of path geometry in a modern framework. Having a spray, one can construct an Ehresmann connection in an intrinsic way, and from this, via linearization, a covariant derivative operator, called Berwald derivative. On the other hand, a Frlicher–Nijenhuis type theory of derivations of differential forms along the tangent bundle projection can be built up (theory of *E. Martínez*, *J. F. Cariñena* and *W. Sarlet*). These tools provide a perfectly adequate and efficient differential calculus for a systematic study of the geometry of paths. In the lecture

I survey some problems and results concerning the different types of the metrizability of a spray; these metrizability problems are closely related to (in particular, coincide with) the inverse problem of calculus of variations.

Masatomo Takahashi (Muroran Institute of Technology, Muroran, Japan)
On implicit ordinary differential equations

`Abstract:` We are going to talk about conditions for existence and uniqueness for geometric solutions of implicit ordinary differential equations.

All manifolds and map germs considered here are differentiable of class C^∞, unless stated otherwise.

An implicit ordinary differential equation is given by the form

$$F\left(x, y, p_1, \ldots, p_n\right) = 0,$$

where F is a smooth function of the independent variable x, of the function y and $p_i = d^i y/dx^i$. It is natural to consider $F = 0$ as being defined on a subset in the space of n-jets of functions of one variable, $F : U \to \mathbb{R}$ where U is an open subset in $J^n(\mathbb{R}, \mathbb{R})$. Throughout this talk, we assume that 0 is a regular value of F.

If $F = 0$ satisfies $\partial F/\partial p_n \neq 0$, by the implicit function theorem, we can locally rewrite this equation in the form $p_n = f(x, y, p_1, \ldots, p_{n-1})$, where f is a smooth function. This explicit form is more convenient than the original one, because there exists the classical existence and uniqueness for (smooth) solutions around a point.

For implicit ordinary differential equations, however, existence and uniqueness for solutions does not hold in general. In this talk, we give a sufficient condition about existence and uniqueness for geometric solutions of implicit ordinary differential equations.

Lajos Tamassy (Debrecen University, Debrecen, Hungary)
Relations between Finsler spaces and distance spaces

`Abstract:` A Finsler space $F^n = (M, \mathcal{F})$ with base manifold M and Finsler metric $\mathcal{F}$ is built on the Finsler norm of infinitesimal vectors, which also means infinitesimal distance. If $c(t) \subset M$ is a curve, then the length (the Finsler norm) $\|\dot{c}(t)\Delta t\|_F$ of the infinitesimal vector $\dot{c}(t)\Delta t$ is measured in the tangent space $T_{c(t)}M$, and the arc length s of $c(t)$ is the integral $\int_a^b \|\dot{c}(t)\|_F$ dt infinitesimal distance. Then the distance $\rho^F(p, q)$, $p, q \in M$ is defined by the infimum of the arc length of the curves $\Gamma(p, q)$ connecting p to q : $\rho^F(p, q) := \inf_{\Gamma(p,q)s}$. (The same is true for Riemann spaces too.) Conversely, a distance space (M, ρ) is built directly on the distance function $\rho : M \times M \to \mathbb{R}^+$, $p, q \mapsto \rho(p, q)$ (satisfying certain simple conditions).

H. Busemann and W. Mayer proved that the distance function ρ^F derived from a Finsler space F^n determines the Finsler space (see also D. Bao - S. S. Chern - Z. Shen). Thus the relation between Finsler spaces $(M, \mathcal{F})$ and distance spaces (M, ρ^F) is $1 : 1$. Nevertheless the family $\{(M, \rho)\}$ of all distance spaces is wider than that of the Finsler spaces $\{(M, \mathcal{F})\}$. This will be shown on concrete examples. In this talk we discuss relations between Finsler and distance spaces. We find necessary conditions in order that a distance space determines a Finsler space. Then we present necessary and sufficient conditions in two versions in

order that (M, ρ) coincides with an $(M, \mathcal{F})$ concerning the distances.

Leonard Todjihounde (Institute of Mathematis and Physics, Porto-Novo, Benin)

Jiri Tomas (Brno University of Technology, Brno, Czech Republic)

Zbynek Urban (Palacky university, Olomouc, Czech Republic)
Higher order Grassmann bundles in mechanics: Variational sequences

`Abstract:` Variational sequences on slide tangent bundles are considered and some variational classes are determined explicitly.

Joris Vankerschaver (Ghent University, Ghent, Belgium)
Symmetry analysis for nonholonomic field theories

Jiri Vanzura (The Academy of Sciences of the Czech Republic, Brno, Czech Republic)
3-forms in dimension 8

Alena Vanzurova (Palacky University, Olomouc, Czech Republic)
On almost geodesic mappings onto generalized Ricci-symmetric manifolds

Petr Vasik (Brno University of Technology, Brno, Czech Republic)
Second order semiholonomic jet transformations

Laszlo Verhoczki (Eotvos Lorand University, Budapest, Hungary)
Cohomogeneity one isometric actions on compact symmetric spaces of type E_6/K

`Abstract:` As is well-known, the exceptional compact Lie group E_6 has four symmetric subgroups up to isomorphisms. These subgroups are $(SU(6) \times SU(2))/\mathbb{Z}_2$, $(Spin(10) \times U(1))/\mathbb{Z}_4$, F_4 and $Sp(4)/\mathbb{Z}_2$. By means of the root space decomposition of the Lie algebra of E_6, we can construct four commuting involutions of the Lie group E_6 which yield these symmetric subgroups. If K is a symmetric subgroup of E_6, then the coset space E_6/K can be equipped with a Riemannian metric such that E_6/K turns into a symmetric space. Three of the four compact Riemannian symmetric spaces of type E_6/K admit isometric actions of cohomogeneity one.

We discuss in detail those cohomogeneity one Hermann actions on these exceptional symmetric spaces, where one of the singular orbits is totally geodesic. Since the principal orbits coincide with tubular hypersurfaces around a singular orbit, the symmetric spaces can be thought of as compact tubes. We determine the radii of the tubes and the shape operators of the principal orbits in terms of the maximal sectional curvature, furthermore, we describe all the orbits as homogeneous spaces. As applications of these results, we compute the volumes of the principal orbits and the volumes of the symmetric spaces. Obviously, the principal orbit of maximal volume is a codimension one minimal submanifold of the ambient space.

Steven Verpoort (Catholic University, Leuven, Belgium)
S. Haesen, **L. Verstraelen**
The Mean Curvature of the Second Fundamental form

`Abstract:` We study semi-Riemannian hypersurfaces of a semi-Riemannian manifold, of which the second fundamental form is an abstract semi-Riemannian metrical tensor. For example, the requirement on a surface in Euclidean space that the second fundamental form is a Riemannian metrical tensor, is equivalent with strict convexity.

In this talk, a study of the critical points of the area functional associated to the second fundamental form is presented. These critical points are characterised by the vanishing of a scalar function, which will be called *the mean curvature of the second fundamental form*.

The mean curvature of the second fundamental form was introduced thirty years ago by E. Glässner and U. Simon for surfaces in Euclidean space. It can easily be seen that the area of the second fundamental form of such a surface can be computed as the integral of the square root of the Gaussian curvature, $\int \sqrt{K}\,\mathrm{d}\Omega$.

On the other hand, the Gaussian curvature of the second fundamental form was already studied by E. Cartan.

Some characterisations of totally umbilical surfaces in a three-dimensional semi-Riemannian manifold, in which the mean curvature of the second fundamental form plays a rôle, are given. A characterisation of Euclidean hyperspheres in terms of the mean curvature of the second fundamental form and the scalar curvature of the second fundamental form is given. This is a generalisation of a result by G. Stamou.

In the particular case of a curve on the unit sphere, the presented results are in agreement with a study by J. Arroyo, O.J. Garay and J.J. Mencía.

Alexandre M. Vinogradov (Universita di Salerno, Salerno, Italy)
Secondary Calculus: New Developments and Results

Luca Vitagliano (Universita degli Studi di Salerno, Salerno, Italy)
On the Geometry of the Covariant Phase Space

`Abstract:` The covariant phase space (CPS) of a lagrangian field system is the solution space of the associated Euler-Lagrange equations. It is in principle a nice environment for covariant quantization of a lagrangian field theory. Indeed, it is manifestly covariant and possesses a canonical (functional) presymplectic structure W (as first noticed by Zuckerman in 1986) whose degeneracy (functional) distribution is naturally interpreted as Lie algebra of gauge transformations. The CPS has been often described by functional analytic methods. In my talk I will describe it (and W) geometrically (and homologically) in the framework of the formal theory of partial differential equations. In particular I will relate W to the first and second Noether theorems by ”computing” its kernel. In the case when W is non degenerate there exist associated (functional) Poisson brackets. These are the so called Peierls brackets of the lagrangian theory.

Raffaele Vitolo (Universita del Salento, Lecce, Italy)
Local variationality through symmetries and conservation laws

`Abstract:` In this talk we consider Yang-Mills-type equations. We first suppose that the number n of independent variables fulfills $n \geq 3$, and that the equations are of second order. Then we show that the existence of translational symmetries and conservation laws, and gauge symmetries and conservation laws, implies that the equations are locally variational. An example shows that higher order Yang-Mills-type equations do not have this property, at least if $n \geq 4$.

Petr Volny (VSB - Technical University of Ostrava, Ostrava, Czech Republic)
Olga Krupkova (Palacky University, Olomouc, Czech Republic)
Jana Volna (Tomas Bata University, Zlin, Czech Republic)
Constrained Lepage forms

`Abstract:` Lepage forms, introduced by D. Krupka in 1973, are known to be fundamental objects in the global calculus of variations on fibred manifolds. We extend this concept to the case of non-holonomic constraints. With help of a constrained Lepage form we construct a constrained Euler–Lagrange form, a global counterpart of variational equations on a constraint submanifold in a jet bundle over a fibred manifold.

Jan Vondra (Masaryk University, Brno, Czech Republic)
Prolongation of general linear connections on W^2P

`Abstract:` Let K is a general linear connection on a vector bundle $E \to M$ and Λ is a linear connection on M. We classify all connections on $W^2PE = P^2M \times_M J^2PE$ naturally given by K and Λ, where PE is the principal bundle of frames of E.

Theodore Voronov (University of Manchester, Manchester, Great Britain)
The Gravitational Field of the Robertson-Walker Spacetime

`Abstract:` We shall discuss geometric and algebraic structures associated with a differential operator of second order acting on functions or on densities (of various weights). Examples include Batalin-Vilkovisky type operators arising in quantum field theory. Particularly interesting cases are the odd Laplace operators acting on half-densities on odd Poisson manifolds, where a groupoid structure of the Batalin-Vilkovisky equation is revealed, and the case of the self-adjoint second-order operators on the algebra of densities on a manifold or supermanifold, which canonically correspond to ”brackets” in this algebra. This algebra possesses a natural ”generalized volume form” and there is a classification of derivations. A bracket in the algebra of densities would contain the following data: an ordinary bracket on functions, a ”contravariant connection” on volume forms and an extra object similar to the Brans-Dicke field in the Kaluza-Klein type theory. There is a nice relation with the appearance of the Schwartz derivative in the transformation law for Sturm-Liouville operators, a well-known fact in integrable systems. On the other hand, passing from the second-order operators to those of higher order leads us to higher brackets and homotopy algebras.

Joanna Welyczko (Wroclaw University of Technology, Wroclaw, Poland)

Ekkehart Winterroth (University of Torino, Torino, Italy)
On complex analytic metrics

`Abstract:` We study the obstructions to the existence of complex analytic metrics on complex manifolds. This is a step towards answering the question whether there exist compact complex manifolds which are not flat and admit a complex analytic metric. Such a manifold would

admit a complex analytic connection and this rules out practically all examples with a well developed theory, that is Kaehler manifolds, especially projective ones, toric varieties, Tits fiberings, etc.

Pit-Man Wong (University of Notre Dame, Notre Dame, Indiana, USA)

Takahiro Yajima (Tohoku University, Sendai, Japan)
Hiroyuki Nagahama
Soliton systems and Zermelo condition in higher-order space

`Abstract:` In this study, we discuss a relation between nonlinear physical fields called soliton systems and a differential geometric space called Kawaguchi space or higher-order space.

In Kawaguchi space, an arc length is given by $ds = F(x^i, x^{(1)i}, \cdots, x^{(M)i})dt$, where $i = 1, 2, \cdots n$. (x^i) is a coordinate system in the n-dimensional configuration space, $x^{(\alpha)i} = d^\alpha x^i/dt^\alpha$ and t is a parameter such as a time. The function F is the Lagrangian of order M. The arc length remains unaltered by a change of the parameter t. Therefore, in Kawaguchi space, the Lagrangian should satisfy the following condition (Zermelo condition):

$$\Delta_k F \equiv \sum_{\alpha=k}^{M} \binom{\alpha}{k} x^{(\alpha-k+1)i} \frac{\partial F}{\partial x^{(\alpha)i}} = \delta_k^1 F, \ k = 1, \cdots M. \tag{1}$$

Geometrically, some field equations in physics can be expressed in terms of Kawaguchi space. For example, Klein-Gordon equation and Dirac equation in quantum field theory have been derived from Zermelo condition. Based on this approach, we show a relation between Kawaguchi space and nonlinear physical systems called soliton systems. In order to obtain a nonlinear expression, we introduce Zermelo operator Δ'_1 in 2-dimensional case as follows

$$\Delta'_1 = \psi\frac{\partial}{\partial\psi} + \psi_i\frac{\partial}{\partial\psi_i} + \psi_{ii}\frac{\partial}{\partial\psi_{ii}} + \cdots, \tag{2}$$

where $\psi = \psi(x,t)$, $\psi_i = \partial\psi/\partial x^i$ and $\psi_{ii} = \partial^2\psi/\partial x^i\partial x^i$. Then, for a certain Lagrangian F, a soliton equation can be obtained from Zermelo condition $\Delta'_1 F = F$. For example, KdV equation $\psi_t + 6\psi\psi_x + \psi_{xxx} = 0$ can be derived from Zermelo condition in Kawaguchi space of order 3. Moreover, a maximum order of differentiation in soliton equations corresponds to the order of Kawaguchi space. Thus, the soliton systems can geometrically connect with Kawaguchi space.

Finally, let us remark that the present approach can apply to a geometric theory of nonlinear dynamical systems called KCC-theory, because Zermelo condition can be regarded as a second variational equation in the KCC-theory. Therefore, the higher-order geometry is also useful in analysis of nonlinear dynamical systems.

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About analogue of fundamental group for semi-submanifold

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The equivalence problem for 2nd-order ODEs

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Symmetries of almost Grassmannian geometries

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Differential geometry of curves in Lagrange Grassmannians with given Young diagram

`Abstract:` Curves in Lagrange Grassmannian appear naturally in the study of geometric structures on a manifold (submanifolds of its tangent bundle). One can consider the time-optimal problem on the set of curves tangent to a geometric structures. Extremals of this optimal problem are integral curves of certain Hamiltonian vector field in the cotangent bundle. The dynamics of the fibers of the cotangent bundle along an extremal w.r.t. to the corresponding Hamiltonian flow is described by certain curve in a Lagrange Grassmannian, called Jacobi curve of the extremal. Any symplectic invariant of the Jacobi curves produces an invariant of the original geometric structure.

The basic characteristic of a curve in a Lagrange Grassmannian is its Young diagram. The number of boxes in its kth column is equal to the rank of the kth derivative of the curve (which is an appropriately defined linear mapping) at a generic point. We will describe the construction of the complete system of symplectic invariants for parameterized curves in a Lagrange Grassmannian with given Young diagram. It allows to develop in a unified way local differential geometry of very wide classes of geometric structures on manifolds, including both classical geometric structures such as Riemannian and Finslerian structures and less classical such as sub-Riemannian or sub-Finslerian structures, defined on nonholonomic distributions. This is joint work with Chengbo Li.

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Complete description of non-Euclid geometry leads to the unification of physics